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Investigating the Relationships between the Yield Curve, Output and Inflation using an Arbitrage-Free Version of the Nelson and Siegel Class of Yield Curve Models

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Abstract

This article provides a theoretical economic foundation for the popular Nelson and Siegel (1987) class of yield curve models (which has been absent up to now). This foundation also offers a new framework for investigating and interpreting the relationships between the yield curve, output, and inflation that have already been well-established empirically in the literature. Specifically, the level of the yield curve as measured by the VAO model is predicted to have a cointegrating relationship with inflation, and the shape of the yield curve as measured by the VAO model is predicted to correspond to the profile (that is, timing and magnitude) of future changes in the output gap (that is, output growth less the growth in potential output). These relationships are confirmed in the empirical analysis on 50 years of United States data.

Keywords

yield curve
term structure of interest rates
Nelson and Siegel model
inflation
output

JEL Classification

E43, E31, E32

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1 Introduction

This article uses the volatility-adjusted orthonormalised Laguerre polynomial (VAO) model of the yield curve derived in Krippner (2005) to investigate the relationship between the yield curve, output, and inflation. The VAO model is an intertemporally-consistent and arbitrage-free version of the Nelson and Siegel (1987) approach to modelling the yield curve, which is used by researchers and market practitioners in a wide variety of markets and applications.[1]

The first contribution of this article is to provide a theoretical economic foundation for the VAO model via an explicit comparison to the generic general equilibrium model of the yield curve proposed by Berardi and Esposito (1999). Because the VAO model incorporates the Nelson and Siegel (1987) model as a special case (as noted in Krippner (2005)), this foundation also provides an economic basis for the Nelson and Siegel (1987) model that has been absent up to now.

The second contribution is to use the VAO model framework to derive the theoretical relationships between the yield curve, output, and inflation. The empirical results using United States data confirm those predicted relationships; i.e the level of the yield curve as measured by the VAO model is cointegrated with inflation, and the shape of the yield curve as measured by the VAO model corresponds to the profile (i.e timing and magnitude) of future changes in the output gap. These results provide an explanation for an extensive body of existing empirically-based literature that establishes, across a wide range of countries, a strong relationship between the current slope of the yield curve and future output growth, and a modest but variable relationship between the current yield curve and future inflation.[2]

Using the VAO model in conjunction with an economic interpretation also complements a growing body of macroeconomic-finance literature that focusses

[1] For example: (1) forecasting the yield curve, Diebold and Li (2002); (2) analysing relative values of fixed interest securities Kacala (1993), and Ioannides (2003); (3) deriving monetary policy expectations Söderlind and Svensson (1997), Monetary Authority of Singapore (1999), and Bank for International Settlements (1999) contains sub-articles and further references regarding ten central banks (of twelve surveyed) that use OLP models; (4) managing fixed interest portfolio risk Barrett, Gosnell and Heuson (1995), Willner (1996), and Diebold and Li (2002); (5) investigating macroeconomic time-series data Diebold, Rudebusch and Aruoba (2003); (6) studying interest rate swap spreads Brooks and Yong Yan (1999), and Fang and Muljono (2003); and (7) providing estimates of zero-coupon yields as a direct valuation exercise or for subsequent empirical analysis Diaz and Skinner (2001), Soto (2001), Schmid and Kalemanova (2002), and Steeley (2004).

[2] For example: (1) on the yield curve/output relationship see Bernanke (1990), Chen (1991) Estrella and Hardouvelis (1991), Davis and Henry (1994), Hardouvelis (1994), Plosser and Rouwenhorst (1994), Dueker (1997), Bernard and Gerlach (1998), Estrella and Mishkin (1998), Filardo (1999), and Atta-Mensah and Tkacz (2001); (2) on the yield curve/inflation relationship see Mishkin (1990*a*), Mishkin (1990*b*), Mishkin (1991), Jorian and Mishkin (1991), Abken (1993), and Schich (1999*a*); and (3) on both the yield curve/output and yield curve/inflation relationships see Gamber (1996), Davis and Fagan (1997), Estrella and Mishkin (1997), Kozicki (1997), Berk (1998), Stock and Watson (2001), and Diebold et al. (2003); and (4) on the stability of the yield curve/output and yield curve/inflation relationships see Haubrich and Dombrosky (1996), Schich (1999*b*), Hamilton and Kim (2002), and Estrella, Rodrigues and Schich (2003).

on why the yield curve/output and yield curve/inflation relationships exist, and also why those relationships might change over time.[3] The general conclusions from that work is that the shape of the yield curve should have a fundamental relationship with expected output and inflation, and that the monetary policy reaction function and central bank credibility (if those are included) play a role in the transmission and therefore the intensity of those relationships. The work in this article is most closely related to two articles: i.e Diebold et al. (2003), which empirically investigates inter-relationships between real economic data, inflation data, and the yield curve using the Nelson and Siegel (1987) model; and de Lint and Stolin (2003), which provides a theoretical basis for the yield curve/output relationship using a model based on an underlying production economy.

The article proceeds as follows: section 2 outlines the elements of the VAO model relevant to this article, and section 3 derives the relationship between the VAO model and the Berardi and Esposito (1999) model of the forward rate curve which allows an interpretation of the VAO model in terms of economic state variables. Section 3 also derives estimable relationships based on those interpretations, and those relationships are investigated empirically in section 4. Section 5 concludes and notes some implications that the work has for the operation of monetary policy.

2 The VAO model of the forward rate curve

Section 2.1 briefly collects the essential assumptions and notation of the generic VAO model of the forward rate curve that is required for deriving the economic foundation in section 3. The full derivation and discussion of the generic VAO model is available in Krippner (2005). Section 2.2 specifies the specific VAO model that used as a practical example throughout this article, and for the empirical work in section 4.

2.1 The generic VAO model of the forward rate curve

The derivation of the VAO model of the forward rate curve is based on the Heath, Jarrow and Morton (1992) (HJM) framework. At each point in time, the HJM framework specifies an intertemporally-consistent and arbitrage-free relationship between: (1) the forward rate curve; (2) the expected path of the short rate; (3) the volatility structure that dictates how the entire forward rate curve can potentially change due to random factors; and (4) the market prices of risk. Defining functional forms for items 2, 3, and 4 therefore defines the functional form for the forward rate curve. The VAO model uses functional forms

[3] For example, Harvey (1988), Hu (1993), Campbell and Cochrane (1999), and Harvey (1997) justify the yield curve/output relationship theoretically using the consumption capital asset pricing model. Smets and Tsatsaronis (1997), Wu (2002), Ang and Piazzesi (2003), de Lint and Stolin (2003), Dewachter and Lyrio (2003), Estrella (2003), Hördahl, Tristani and Vestin (2003), and Rudebusch and Wu (2003) investigate the relationships using multi-variate structural models that include interest rates or the yield curve in conjunction with other financial and macroeconomic variables, and typically a monetary policy reaction function.

analogous to the Nelson and Siegel (1987) approach, i.e exponential-polynomial or orthonormalised Laguerre polynomial (OLP) functions, as specified in Definition 1.

Definition 1 ***The generic VAO model of the forward rate curve***

Assumption 1: At time t and as a function of future time $t+m$ ($m \geq 0$), the expected path of the short rate $E_t[r(t+m)]$ under the physical measure is defined as:

$$E_t[r(t+m)] = \sum_{n=1}^{N} \lambda_n(t) \cdot g_n(\phi, m) \tag{1}$$

where E_t is the expectations operator as at time t; $\lambda_n(t)$ are time-varying coefficients, and $g_n(\phi, m)$ are the short rate modes defined as $g_1(\phi, m) = 1$, and for $n > 1$:

$$g_n(\phi, m) = -\exp(-\phi m) \cdot \sum_{k=0}^{n-2} \frac{(-1)^k (n-2)!(2\phi m)^k}{(k!)^2 (n-2-k)!} \tag{2}$$

Assumption 2: Potential stochastic changes to the expected path of the short rate, i.e $d\{E_t[r(t+m)]\}_{Stoc.}$, are defined as:

$$d\{E_t[r(t+m)]\}_{Stoc.} = \sum_{n=1}^{N} \sigma_n \cdot g_n(\phi, m) \cdot dW_n(t+m) \tag{3}$$

where σ_n are constant standard deviations, and $dW_n(t+m)$ are Wiener increments under the physical measure.

Assumption 3: The expected market prices of risk associated with each mode, i.e θ_n, are constants.

Then, at time t as a function of maturity m, the forward rate curve $f(t,m)$ under the physical measure will have the following functional form:

$$f(t,m) = \sigma_1\theta_1 m + \sum_{n=1}^{N} \beta_n(t) \cdot g_n(\phi, m) - \sum_{n=1}^{N} \sigma_n^2 \cdot h_n(\phi, m) \tag{4}$$

where $\beta_n(t) = \gamma_n + \lambda_n(t)$, γ_n are constant parameters each expressible as linear combinations of $\sigma_1\theta_1, \sigma_2\theta_2, \ldots, \sigma_N\theta_N$, and $h_n(\phi, m)$ are time-invariant functions of maturity that may be derived as:

$$h_n(\phi, m) = \frac{1}{2\phi^2} \cdot \sum_{k=0}^{n-2} \frac{(-2)^k (n-2)!}{(k!)^2 (n-2-k)!} \cdot (k! - \Gamma[1+k, \phi m])^2 \tag{5}$$

where $\Gamma[\cdot,\cdot]$ is the incomplete Gamma function.

2.2 The VAO(3) model used in the empirical work

The practical discussion and the empirical work in this article uses the $N = 3$ version of the VAO model for the forward rate curve, or the VAO(3) model for short. Hence, $E_t[r(t+m)]$ is represented as a linear combination of the

three short rate modes $g_1(\phi, m) = 1$, $g_2(\phi, m) = -\exp(-\phi m)$, and $g_3(\phi, m) = -\exp(-\phi m)(-2\phi m + 1)$. These three modes are illustrated in figure 1, and are colloquially named the Level, Slope and Bow modes in reference to their intuitive shapes.[4] The estimation of the VAO model coefficients and parameters from from market-quoted interest rate data is discussed in section 4.1.

[Figure 1 here]

3 An economic interpretation of the VAO model

Section 3 proceeds as follows: section 3.1 summarises a generic general equilibrium economy approach to modelling the yield curve proposed by Berardi and Esposito (1999), and sections 3.2 and 3.3 respectively show that the real and inflation components of the expected path of the short rate from that model are naturally approximated by OLP modes used to represent the expected path of the short rate in the VAO model. Using these results, section 3.4 discusses the economic interpretation of the generic VAO model parameters and coefficients. Finally, section 3.5 discusses how a model defined in instantaneous time may be converted into estimable relationships for data measured over discrete time periods.

3.1 A generic general equilibrium approach to modelling the yield curve

Berardi and Esposito (1999) (hereafter BE) derives a generic affine multifactor model of the forward rate curve from a general equilibrium model based on the economic model proposed by Cox, Ingersoll and Ross (1985*a*). The BE approach encompasses all Vasicek-type and Cox-Ingersoll-Ross-type equilibrium models,[5] and many other equilibrium models that have been proposed in the literature. It also encompasses the affine multifactor models of Duffie and Kan (1996) and Dai and Singleton (2000), providing a general equilibrium basis for those models and explicitly accounting for the separation between real and nominal variables. The BE generic J-factor process under the physical measure is:

$$ds_j(t) = -\kappa_j [s_j(t) - \theta_j]\, dt + \sqrt{\sigma_{0j}^2 + \sigma_{1j}^2 \cdot s_j(t)} \cdot dz_j(t) \tag{6}$$

where, for $j = 2$ to J, $s_j(t)$ are the real state variables, representing instantaneous returns on factors of production in the economy (these will change

[4] Note that, for the advantage of comparability, the VAO(3) model is analogous to the Nelson and Siegel (1987) model of the forward rate curve (which uses three linear coefficients applied to the first three OLP functions to represent the forward rate curve), and a three-coefficient model is also consistent with the idea that three principal components may be used to adequately capture almost all of the variation in the yield curve over time (as suggested in Litterman and Sheinkman (1991)).

[5] That is, Gaussian and square root dynamics, respectively. See the original article, Vasicek (1977), or Hull (2000) p. 567 for a summary of the Vasicek equilibrium model, and the original article, Cox, Ingersoll and Ross (1985*b*), or Hull (2000) p. 570 for a summary of the Cox-Ingersoll-Ross equilibrium model.

with a deterministic and stochastic component as time evolves, and are constructed from the original state variables so that all innovations are mutually uncorrelated); κ_j (> 0) is the constant mean-reversion coefficient of the process for $s_j(t)$; θ_j (> 0) is the constant steady-state (long-run) value of $s_j(t)$; $\sqrt{\sigma_{0j}^2 + \sigma_{1j}^2 \cdot s_j(t)}$ is the standard deviation of the stochastic process for $s_j(t)$;[6] and $dz_j(t)$ are independent Wiener variables under the physical measure.

The $j = 1$ factor is reserved for an inflation state variable, which will be discussed in section 3.3. As noted in BE, the nominal short rate at any given time is the summation of state variables $s_j(t)$, and for the analysis that follows it is convenient to partition this into inflation and real components, i.e $r(t, s_1, \mathbf{s}) = s_1(t) + \sum_{j=2}^{J} s_j(t)$, where $r(t, s_1, \mathbf{s})$ is the instantaneous nominal short rate as a function of the instantaneous value of the inflation state variable $s_1(t)$ and the $(J-1)$-vector of real state variables $\mathbf{s}(t)$, and $\mathbf{s}(t)$ contributes the instantaneous real interest rate component $\sum_{j=2}^{J} s_j(t)$.

3.2 The real components of the BE model

The expected path of the real short rate may be calculated directly from the expectation of equation 6. That is, applying the expectations operator at time t and using m to denote future time from time t gives the relationship: $E_t[ds_j(t+m)] = -\kappa_j[s_j(t+m) - \theta_j]\,dm$. This is an ordinary differential equation with solution $E_t[s_j(t+m)] = \theta_j + A_j \cdot \exp(-\kappa_j m)$, and the boundary condition at $m = 0$ is $s_j(t) = \theta_j + A_j$, so $A_j = s_j(t) - \theta_j$. Therefore, the real component of the expected path of the short rate may be written as:[7]

$$\sum_{j=2}^{J} E_t[s_j(t+m)] = \sum_{j=2}^{J} \theta_j + \sum_{j=2}^{J} [s_j(t) - \theta_j] \cdot \exp(-\kappa_j m) \tag{7}$$

To show the correspondence between equation 7 and the OLP functional form in equations 1 and 2 that is used to represent the expected path of the short rate within the VAO model, first define ϕ as a central measure of the values of κ_j for $j = 2$ to J, i.e $\phi = \text{central}(\kappa_j)$ (which is a constant, because κ_j are constants). Hence, $\kappa_j = \phi(1 + \Delta_j)$ with $-1 < \Delta_j < 1$,[8] and equation 7 may be written equivalently as:

$$\sum_{j=2}^{J} E_t[s_j(t+m)] = \sum_{j=2}^{J} \theta_j + \exp(-\phi m) \cdot \sum_{j=2}^{J} [s_j(t) - \theta_j] \cdot \exp(-\Delta_j \phi m) \tag{8}$$

[6] The process will be Vasicek-type if $\sigma_{1j} = 0$, Cox-Ingersoll-Ross-type if $\sigma_{0j} = 0$, and can be a mixture of both if σ_{0j} and σ_{1j} are non-zero (with appropriate restrictions to keep $\sigma_{0j}^2 + \sigma_{1j}^2 \cdot s_j(t)$ positive).

[7] This result, and the analogous result for the inflation component in section 3.3, can also be derived using the forward rate curve specified by BE, and calculating the associated expected path of the short rate using the HJM framework.

[8] This restriction on Δ_j is always possible by construction; in the extreme case, ϕ could be defined as $\max(\kappa_j)$, and then $-1 < \Delta_j \leq 0 < 1$ (because the lower bound for each κ_j is zero).

Now write each exponential term containing Δ_j as a Taylor expansion around $\Delta_j = 0$ to order $N-2$; i.e $\sum_{j=2}^{J} E_t\left[s_j\left(t+m\right)\right]$ may be approximated to arbitrary precision as:[9]

$$\sum_{j=2}^{J}\theta_j + \exp\left(-\phi m\right)\cdot\sum_{j=2}^{J}\left[s_j\left(t\right)-\theta_j\right]\left[\sum_{n=2}^{N}\frac{1}{(n-2)!}\left(-\Delta_j\phi m\right)^{(n-2)}\right] \tag{9a}$$

$$= \sum_{j=2}^{J}\theta_j + \exp\left(-\phi m\right)\cdot\sum_{n=2}^{N}\omega_n\left(t\right)\cdot\left(\phi m\right)^{(n-2)} \tag{9b}$$

$$= \sum_{j=2}^{J}\theta_j - \sum_{n=2}^{N}\lambda_n\left(t\right)\cdot -\exp\left(-\phi m\right)\sum_{k=0}^{n-2}\frac{(-1)^k\,(n-2)!(2\phi m)^k}{\left(k!\right)^2\,(n-2-k)!} \tag{9c}$$

where the coefficients $\omega_n\left(t\right)$ in equation 9b are the collections of the coefficients on powers of $\left(\phi m\right)^{(n-2)}$ from the full expansion of the double summation in equation 9a, and equation 9c is a rearrangement of the summation of exponential-polynomials into a linearly equivalent summation of OLP functions. This is the generic OLP form noted in equations 1 and 2.

3.3 The inflation component of the BE model

BE uses a single independent factor to represent the instantaneous rate of inflation in the general equilibrium model. For this factor, each of the parameters in equation 6 are analogous to their real counterparts.[10] However, the BE inflation factor has an important analytical difference to the real factors discussed in section 3.2, because the mean-reversion coefficient κ_1 is much smaller than for the real factors. Weakly mean-reverting inflation is consistent with the general macroeconomic notion of inflation persistence, and also with the Fisher hypothesis that changes in nominal long-maturity rates are determined almost exclusively by changes to the expected inflation rate.[11] In addition, empirical estimates of κ_1 from BE and Brown and Schaefer (1994) confirm that mean-reversion in long time-series of inflation data has typically been low enough to be insignificantly different from zero.

Weakly mean-reverting inflation may be approximated by equation 6 with $\kappa_1 = 0$, i.e $ds_1\left(t\right) = \sqrt{\sigma_{0,1}^2 + \sigma_{1,1}^2\cdot s_1\left(t\right)}\cdot dz_j\left(t\right)$. Applying the expectations operator at time t and using m to denote future time from time t gives a trivial ordinary differential equation $E_t\left[ds_1\left(t+m\right)\right] = 0$. This has the solution

[9] The residual term $\sum_{n=N+1}^{\infty}\frac{1}{(n-2)!}\left(-\Delta_j\phi m\right)^{(n-2)}$ associated with the Taylor expansion approximation will always converge to a finite value, which may be made arbitrarily small, because $\left|\Delta_j\right| < 1$.

[10] Some parameters are a combination of the relative price level and inflation rate parameters. Specifically, $s_1\left(t\right) = \pi\left(t\right) - \sigma_p^2$; $\kappa_1 = \kappa_\pi$; $\theta_1 = \theta_\pi - \sigma_p^2$; and $\sigma_1 = \sigma_\pi$, where $\pi\left(t\right)$ is the inflation rate, σ_p^2 is the variance of relative changes in the price level, κ_π is the mean-reversion coefficient for the inflation rate, θ_π is the long-term inflation rate, and σ_π is the standard deviation of the inflation rate.

[11] See, for example, Walsh (1998) pp. 215-226 and pp. 345-351 regarding inflation persistence, and p. 459 regarding the Fisher hypothesis.

$E_t[s_1(t+m)] = A_1$, the boundary condition at $m = 0$ is $s_1(t) = A_1$, and so $E_t[s_1(t+m)] = s_1(t)$.[12] The inflation component of the expected short rate in the BE model assuming zero mean-reversion is therefore a constant by maturity, and so expected inflation over short horizons will equal expected inflation over long horizons. This will not always be the case in practice (particularly in transition eras of rising inflation or disinflation, or strict inflation targeting regimes), but it should be a reasonable approximation for long data series, including the 50 year sample of United States data in the empirical application of section 4.[13]

One potential over-simplification of the BE model is that the inflation factor is assumed to be independent of the real factors. If there is a relationship between the output gap and inflation, and/or the central bank broadly sets the short-term interest rate in response to its assessment of those variables and its policy goals,[14] then the overall relationship between interest rates, the yield curve, output, and inflation is likely to be more complex. This aspect is discussed in section 3.4.5.

3.4 The economic interpretation of the VAO model

Denoting the expected paths of the short rate within the BE and the VAO models as BE$\{E_t[r(t+m)]\}$ and VAO$\{E_t[r(t+m)]\}$, the key results from sections 3.2 and 3.3, and the relationship to the VAO model in section 2.1 may be summarised as follows:

[12] $s_1(t)$ is therefore a random-walk. However, $\kappa_1 = 0$ is not an isolated special case, but is rather the natural limit of a small but finite κ_1 that would be consistent with a near-random-walk; i.e as $\kappa_1 \to 0$, $E_t[s_1(t+m)] = \theta_j + [s_1(t) - \theta_j] \cdot \exp(-\kappa_1 m) \to s_1(t)$.

[13] Indeed, even recently, the 1–year and 10–year expected average CPI inflation data from the Survey of Professional Forecasters have typically been very close and homogeneous. Specifically, using the 51 quarterly observations available since December 1991, the average difference between the 1-year and 10-year expectations is 17 basis points, with a standard deviation, minimum, and maximum of 21, -19 and 77 basis points respectively. Unit root tests indicate that the two series are cointegrated with vector (1,-1); i.e the Dickey-Fuller statistic is -1.31 for the 1-year, -1.66 for the 10 year, and -2.73 for the difference (where the 5 percent critical level is -1.95).

[14] See, for example, Romer (2001) pp. 245-252, and pp. 500-503 for discussion on these respective issues.

$$\begin{aligned}
\mathrm{BE}\left\{E_t\left[r(t+m)\right]\right\} &= \sum_{j=1}^{J} E_t\left[s_j\left(t+m\right)\right] && \text{(10a)}\\
&= E_t\left[s_1\left(t+m\right)\right] + \sum_{j=2}^{J}\theta_j \\
&\quad + \sum_{j=2}^{J} E_t\left[s_j\left(t+m\right)\right] - \theta_j && \text{(10b)}\\
&= s_1\left(t\right) + \sum_{j=2}^{J}\theta_j + \sum_{j=2}^{J}\left[s_j\left(t\right) - \theta_j\right]\cdot\exp\left(-\kappa_j m\right) && \text{(10c)}\\
&\simeq \lambda_1\left(t\right) - \sum_{n=2}^{N}\lambda_n\left(t\right)\cdot g_n(\phi, m) && \text{(10d)}\\
&= \mathrm{VAO}\left\{E_t\left[r(t+m)\right]\right\} && \text{(10e)}
\end{aligned}$$

Equation 10 firstly shows that the generic N-mode expected path of the short rate in the VAO model is a natural approximation to the BE model of expected path of the short rate with J state variables. In other words, the VAO model of the forward rate curve (and yield curve) uses N coefficients to approximate the J-state-variable BE model of the forward rate curve (and yield curve). The approximation is natural in the sense that each additional mode in the VAO model beyond the Level mode represents an extra term in the Taylor series expansion from the BE model of the expected path of the short rate.[15] The VAO(3) model specified and used for the empirical work in this article is therefore a second-order Taylor approximation to the BE model.

Secondly, because $\mathrm{BE}\{E_t\left[r(t+m)\right]\}$ has an explicit basis in the underlying economic state variables and parameters of the BE general equilibrium economy, equation 10 implies an economic interpretation for the VAO model coefficients and parameters. Also, because the VAO model incorporates the Nelson and Siegel (1987) model as a special case (as noted in Krippner (2005)), this approach provides an economic basis for the Nelson and Siegel (1987) model that has been absent up to now.

Before outlining those economic interpretations, it is convenient to introduce some macroeconomic terminology for three aggregate concepts from the BE model, i.e: (1) $dY\left(t\right) = \sum_{j=2}^{J} s_j\left(t\right)$, where $dY\left(t\right)$ is real instantaneous output growth (i.e the sum of the instantaneous growth rates in each factor of production); (2) $dY^* = \sum_{j=2}^{J}\theta_j$, where dY^* is the steady-state or potential real instantaneous output growth; and (3) $dX\left(t\right) = dY\left(t\right) - dY^* = \sum_{j=2}^{J}\left[s_j\left(t\right) - \theta_j\right]$, where $dX\left(t\right)$ is the instantaneous change in the output gap. Hence, the output gap $X\left(t\right)$ will become less negative or more positive when $dX\left(t\right)$ is positive

[15] Approximating the BE model using simple polynomials (as in McCulloch (1971)), or other families of orthogonal polynomials (such as Chebyshev polynomials, as in Pham (1998)) would be "unnatural" in the sense that the addition of each higher function would not directly correspond to an extra term in the Taylor expansion.

$(dY(t) > dY^*)$, and will become less positive or more negative when $dX(t)$ is negative $(dY(t) < dY^*)$.

Note that this macroeconomic terminology exposes another potential over-simplification of the BE model; i.e dY^* is implicitly constant over time, whereas in practice it might be expected to vary with changes in productivity growth. This flexibility can easily be incorporated in a BE-type model where dY^* is allowed to have periodic and unanticipated changes over time (a reasonable assumption if productivity "shocks" are responsible for the time-variation in dY^*), so that $E[dY^*(t+m)] = dY^*(t)$. The exposition continues with a constant dY^* for simplicity of exposition, but the effects of time-varying dY^* are noted in the relevant sections and the empirical application in section 4 also investigates the case where the data is derived with time-varying dY^*.

3.4.1 The VAO Level coefficient, $\beta_1(t)$

The first economic interpretation from equation 10 is the relationship between the VAO Level coefficient and expected inflation. That is, $E_t[s_1(t+m)] + dY^* \simeq \lambda_1(t)$, and so:

$$\beta_1(t) \simeq \gamma_1 + E_t[s_1(t+m)] + dY^* \tag{11}$$

Hence, $\beta_1(t)$ may be interpreted as the combination of the VAO Level term premium component γ_1, the expected rate of inflation $E_t[s_1(t+m)]$, and the growth in potential output dY^*. If both γ_1 and dY^* were truly constant over time, then $\beta_1(t)$ would have a strictly homogeneous correspondence with the inflation rate, i.e $\beta_1(t)$ would always be within a constant of the $s_1(t)$, and stochastic changes to $s_1(t)$ (i.e inflation "shocks") would be reflected identically as stochastic changes to $\beta_1(t)$. However, if γ_1 and/or dY^* have time-varying and/or structural change components, as is likely in practice, then the relationship between the VAO Level coefficient and the rate of inflation may not be strictly homogeneous. This is discussed further in section 3.5.2.

3.4.2 The non-Level VAO coefficients, $\beta_2(t)$, $\beta_3(t)$, ..., $\beta_N(t)$

The second economic interpretation from equation 10 is the relationship between the non-Level VAO coefficients and the expected change in the output gap. That is, $\sum_{j=1}^{J} E_t[s_j(t+m)] - \theta_j = dY(t+m) - dY^* = E_t[dX(t+m)] \simeq -\sum_{n=2}^{N} \lambda_n(t) \cdot g_n(\phi, m)$, so:

$$\sum_{n=2}^{N} \beta_n(t) \cdot g_n(\phi, m) \simeq \sum_{n=2}^{N} \gamma_n \cdot g_n(\phi, m) - E_t[dX(t+m)] \tag{12}$$

Hence, the current shape of the yield curve as summarised by non-Level VAO coefficients implies an expectation about the profile of future changes in the output gap. For example, at $m = 0$, $g_n(\phi, 0) = -1$ for $n \geq 2$ and $\exp(-\kappa_j \cdot 0) = 1$, so equations 10c and 10d give $\sum_{n=2}^{N} \beta_n(t) = dX(t) + \sum_{n=2}^{N} \gamma_n$. Hence,

when the sum of the non-Level VAO coefficients is positive (i.e a positively-sloped yield curve), this implies that the expected instantaneous change in the current output gap is positive.

Stochastic changes to the non-Level VAO coefficients therefore imply unanticipated changes to the expected profile of the change in the output gap. Or alternatively, "shocks" to the real economy should result in changes to the shape of the yield curve, which will be reflected as stochastic changes to the non-Level coefficients of the VAO model.

3.4.3 The VAO model exponential decay parameter, ϕ

The third economic interpretation from equation 10 and section 3.2 is that the exponential decay parameter ϕ in the VAO model may be interpreted as a central measure of the mean-reversion coefficients of the real state variable processes in the BE model, i.e $\phi = \text{central}(\kappa_j)$ (which is a constant, because κ_j are constants). Hence, "shocks" to the growth rates of individual factors of production relative to their steady-state growth rates should typically persist with a decay rate of ϕ, i.e a half-life of $\ln(2)/\phi$. And because $dX(t)$ is an aggregate of the growth rates for all factors of production relative to their steady-state growth rates, the change in the output gap should also have a half-life of approximately $\ln(2)/\phi$.[16]

3.4.4 The number of modes in the VAO model, N

The fourth economic interpretation from equation 10 and section 3.2 is that the empirical significance of higher-order modes in the VAO model should indicate the relative distribution of Δ_j, i.e the magnitudes of the mean-reversion coefficients for the real state variables κ_j relative to central(κ_j). If higher-order modes in the VAO model quickly become empirically insignificant, this would suggest that the magnitudes of κ_j are generally similar, and/or that factors of production with materially different κ_j form a relatively small proportion of the economy.[17]

3.4.5 The covariance structure of the VAO model

The BE model assumes zero covariance between inflation innovations and real factor innovations. This corresponds with the VAO model assumption of zero covariance between stochastic changes in $\beta_1(t)$ (which represents inflation) and the remaining $\beta_n(t)$ coefficients (which represent real factors). However, as noted at the end of section 3.3, the potential interactions between inflation and

[16] As an aside, ϕ could be interpreted as an empirical measure of the "flexibility" of the economy. Hence, it may be worthwhile investigating in future work whether ϕ differs between economies.

[17] The empirical success of three-mode OLP models in many different markets suggests that one or both of these conditions generally hold. However, it may be worthwhile specifically investigating the empirical significance of higher-order modes in the VAO model in future work.

the real economy are likely to be more complex, and so the BE and VAO model assumptions of zero innovation covariances might not be realised empirically.

That said, rather than trying to pre-specify and embed extra theoretical relationships in the model, as in the work of Ang and Piazzesi (2003), Estrella (2003), and Hördahl et al. (2003), the VAO model is deliberately left its original form here. This will allow an empirical investigation of the significance of potentially time-varying relationships between inflation and output growth innovations from a market perspective (i.e by statistically testing for non-zero covariances). For example, a strongly positive covariance might imply a market sensitivity that surprises on output growth would translate directly to higher inflation (or vice-versa). In addition, non-zero covariances could easily be incorporated into an innovation-orthoganalised version of the VAO model. Both of these aspects are left for future work.

3.5 Estimable relationships for the VAO(3) model coefficients, output growth, and expected inflation

While sections 3.4.1 and 3.4.2 suggest potential relationships between the coefficients of the VAO(3) model, instantaneous expected inflation, and instantaneous changes in the output gap, an obvious practical issue is that the latter data are not available. That is, output and price data are only measured at periodic intervals, and so any associated changes will relate to averages of instantaneous changes over the given period. Hence, the following sections convert the continuous-time relationships of the previous section into periodic relationships that are estimable using the available data.

3.5.1 The relationship between the change in the output gap and the VAO(3) Slope and Bow coefficients

The average of $E_t\left[dX\left(t+m\right)\right]$ over a given period of time may be calculated by direct integration; i.e $E_t\left[\Delta X_{t+T_1,t+T_2}\right] = \frac{1}{T_2-T_1}\int_{T_1}^{T_2} E_t\left[dX\left(t+m\right)\right]dm$, where $E_t\left[\Delta X_{t+T_1,t+T_2}\right]$ is, as at time t, the expected change in the output gap between the times $t+T_1$ and $t+T_2$, and T_1 and T_2 represent a forward horizon from time t (i.e $t \leq T_1 < T_2$). Substituting for $E_t\left[dX\left(t+m\right)\right]$ from equation 12 gives the following result for $E_t\left[\Delta X_{t+T_1,t+T_2}\right]$:

$$\frac{-1}{T_2-T_1}\int_{T_1}^{T_2}\left[\sum_{n=2}^{N}\left[\beta_n\left(t\right)-\gamma_n\right]\cdot g_n(\phi,m)\right]dm \tag{13a}$$

$$= \sum_{n=2}^{N}\left[\beta_n\left(t\right)-\gamma_n\right]\cdot\frac{-1}{T_2-T_1}\int_{T_1}^{T_2} g_n(\phi,m)dm \tag{13b}$$

$$= -\sum_{n=2}^{N}\gamma_n\cdot q_n\left(T_1,T_2\right)+\sum_{n=2}^{N}\beta_n\left(t\right)\cdot q_n\left(T_1,T_2\right) \tag{13c}$$

where $q_n(T_1,T_2) = \frac{-1}{T_2-T_1}\int_{T_1}^{T_2} g_n(\phi,m)dm$. The two integrals required for the VAO(3) model are:

$$q_2(T_1,T_2) = \frac{-1}{\phi(T_2-T_1)}\left[\exp(-\phi T_2) - \exp(-\phi T_1)\right] \quad (14a)$$

$$q_3(T_1,T_2) = \frac{-1}{\phi(T_2-T_1)}\left[\begin{array}{r}\exp(-\phi T_2)(-2\phi T_2-1) \\ -\exp(-\phi T_1)(-2\phi T_1-1)\end{array}\right] \quad (14b)$$

Table 1 contains the values of $q_2(T_1,T_2)$ and $q_3(T_1,T_2)$ that correspond to the forward horizons tested in the empirical work. For example, using the Slope and Bow coefficients of the VAO(3) model estimated at time t, the expectation of the change in the output gap between times $t+1$ year and $t+2$ years (i.e $E_t[\Delta X_{t+1,t+2}]$) would be $-\sum_{n=2}^{3}\gamma_n \cdot q_n(1,2) + \sum_{n=2}^{N}\beta_n(t)\cdot q_n(1,2) = -\gamma_1 \cdot 0.21 + -\gamma_2 \cdot -0.43 + \beta_2(t)\cdot 0.21 + \beta_3(t)\cdot -0.43$.

Because the values of γ_2 and γ_3 are constant in the VAO model, for each forward horizon $-\sum_{n=2}^{N}\gamma_n \cdot q_n(T_1,T_2)$ will be a constant, which may be denoted as α_{0,T_1,T_2}. Then using the assumption that market expectations are formed rationally, the difference between the expected and the actual change in the output gap, i.e $\varepsilon_{T_1,T_2} = \Delta X_{t+T_1,t+T_2} - E_t[\Delta X_{t+T_1,t+T_2}]$, should be orthogonal to $E_t[\Delta X_{t+T_1,t+T_2}]$ (this assumption is standard in the literature, e.g see Estrella et al. (2003)). Hence, the relationship for each forward horizon becomes:

$$\Delta X_{t+T_1,t+T_2} = \alpha_{0,T_1,T_2} + \alpha_{1,T_1,T_2}\cdot\sum_{n=2}^{3}\beta_n(t)\cdot q_n(T_1,T_2) + \varepsilon_{T_1,T_2} \quad (15)$$

which is estimable using available data. Note that ε_{T_1,T_2} will have expected moving-average serial correlation when the horizon T_2 exceeds the frequency of observations, and additional moving-average serial correlation will be expected when annual data is used at quarterly frequencies. Hence, the estimated standard errors of the coefficients in equation 15 are calculated using the Newey-West technique with the appropriate window, which allows for the expected autocorrelation and also potential heteroscedasticity in ε_{T_1,T_2} (this technique is standard in this literature, e.g. see Estrella et al. (2003)).

The results expected from the estimation of equation 15 are: (1) α_{0,T_1,T_2} should be negative if term premia are positive (i.e the yield curve would persistently over-forecast the realised change in the output gap, so a negative adjustment is required to remove that bias), and the magnitudes should decline by forward horizon given the declining magnitudes of $q_n(T_1,T_2)$; (2) α_{1,T_1,T_2} should be 1 if the economic interpretations of the VAO model framework are valid; and (3) the explanatory power of the regression should decline by forward horizon as the strength of the $\sum_{n=2}^{3}\beta_n(t)\cdot q_n(T_1,T_2)$ "signal" reduces (due to the falling magnitudes of $q_n(T_1,T_2)$ by forward horizon) and the ε_{T_1,T_2} "noise" increases (due to the aggregation of more expectational surprises).

3.5.2 The relationship between inflation and the VAO(3) Level coefficient

The average of $E_t[s_1(t+m)]$ over a given period of time may also be be calculated by direct integration; i.e $E_t[\pi_{t+T_1,t+T_2}] = \frac{1}{T_2-T_1}\int_{T_1}^{T_2} E_t[s_1(t+m)]\,dm$, where $E_t[\pi_{t+T_1,t+T_2}]$ is the expected value, as at time t, of the inflation rate between the times $t+T_1$, and $t+T_2$. Substituting for $E_t[s_1(t+m)]$ from equation 11 gives the result:

$$E_t[\pi_{t+T_1,t+T_2}] = \frac{1}{T_2-T_1}\int_{T_1}^{T_2}[\beta_1(t) - \gamma_1 - dY^*]\,dm \tag{16a}$$

$$= \beta_1(t) - \gamma_1 - dY^* \tag{16b}$$

Collecting γ_1 and dY^* into the constant α_{π,T_1,T_2}, and again assuming that expectations are formed rationally, the relationship for each forward horizon becomes:

$$\beta_1(t) - \pi_{t+T_1,t+T_2} = \alpha_{\pi,T_1,T_2} + \varepsilon_{\pi,T_1,T_2} \tag{17}$$

which is an estimable relationship using available inflation or inflation expectations data. While equation 17 is similar in form and construction to 15, the difference is that Krippner (2005) shows that $\beta_1(t)$ should follow a random walk (or a near random walk), and section 3.3 notes that measures of inflation have typically been weakly mean-reverting. Hence, equation 17 represents a potential homogeneous cointegrating relationship (i.e a (1,-1) vector) between the VAO Level coefficient and measures of inflation or expected inflation; i.e the Level coefficient series and inflation data should not reject the unit root hypothesis, but the difference should reject the unit root hypothesis.

As noted in section 3.4.1, potential complications in the expected cointegrating relationship will arise if γ_1 and dY^* are not truly constant over time. If γ_1 and dY^* have relatively small variances and/or structural shifts compared to the measure of expected inflation, then the expected homogeneous cointegrating relationship should not be rejected. However, large variances and/or structural changes may contaminate the expected relationship, which would require an augmented version of equation 17. This is discussed further in section 4.

4 Empirical tests

Following the majority of the existing literature in this area, the empirical work in this article is in-sample analysis only; i.e investigating whether the relationships exist as predicted using the full sample of available data. Out-of-sample analysis, which is important for assessing how much reliance may be placed on the relationships in real-time, is left for future work. Section 4.1 outlines the data used in the empirical work, and sections 4.2 and 4.3 contain the empirical results.

4.1 The data

4.1.1 The interest rate data and the VAO(3) model coefficient estimates

The interest rate data used in the empirical application are monthly averages of constant maturity bond rates obtained from the online Federal Reserve Economic Database (FRED) available on the Federal Reserve Bank of St Louis website. The specific series are the federal funds rate, the 3-month Treasury bill rate, and the 1-year, 3-year, 5-year, 10-year, and 20-year or 30-year constant maturity bond rates.[18] The sample period is July 1954 (the first month federal funds rate data is available) to May 2004 (the last month available at the time of the analysis), giving 599 monthly observations of the yield curve. Figure 2 plots the 3-month and 10-year interest rate data, and the difference between these two rates.

[Figure 2 here]

The method used to estimate the VAO(3) model coefficients for each cross-section of yield curve data is detailed in Appendix C of Krippner (2005). As an example, figure 3 illustrates the intuition and the results of the cross-sectional estimation process using the yield curve data from May 2004.[19]

[Figure 3 here]

Each monthly observation of yield curve data will give an associated estimate of the Level, Slope, and Bow coefficients for that month. Hence, the full sample of yield curve data is processed into the three time series, i.e $\beta_1(t)$, $\beta_3(t)$, and $\beta_3(t)$, each containing 599 monthly observations. These time-series are illustrated in figure 4. Comparing these to the original data, it is apparent that the Level series broadly corresponds to the 10-year rate series in figure 2, and the Slope series broadly corresponds to the 10-year less 3-month rate spread (which is typically used as an indicator of the slope of the yield curve in this literature).

[Figure 4 here]

4.1.2 Measures of the change in the output gap

Two measures of the change in the output gap are tested in the estimation of equation 15. The first measure is based on the simplifying assumption that the growth rate in potential output is constant (so the level of potential output is a linear time trend). Hence, the constant potential growth rate is first calculated as the average annualised quarterly GDP growth over the entire sample, i.e $Y^* = \frac{4}{199}\sum_{t=2}^{200}\left[\ln\left(\mathrm{GDP}_t\right) - \ln\left(\mathrm{GDP}_{t-0.25}\right)\right]$, where GDP_t is the the level of GDP as obtained from the FRED. The forward change in the output gap is then constructed by subtracting Y^* from actual GDP growth over the appropriate

[18] 20-year data is unavailable from January 1987 to September 1993, and so 30-year data (with a 30-year maturity) is used during this period for the estimation.

[19] The cross-section estimation process also requires the parameters ϕ, θ_1, σ_1, σ_2, and σ_3. These are estimated using the entire sample of yield curve data, following the process noted in Appendix C of Krippner (2005). The resulting values are ϕ=1.07, θ_1=2.57%, σ_1=0.79%, σ_2=2.31%, and σ_3=1.78%.

forward horizon; i.e $\Delta X_{t+T_1,t+T_2} = \Delta Y_{t+T_1,t+T_2} - Y^*$, where $\Delta Y_{t+T_1,t+T_2} = \ln(\text{GDP}_{t+T_2}) - \ln(\text{GDP}_{t+T_1})$. One advantage of using this restrictive measure is that, to within a constant, the estimation of equation 15 will be directly analogous to the prior empirical literature (as noted in section 1) that regresses forward output growth on the slope of the yield curve as measured by interest rate spreads.

The second measure of the change in the output gap is based on the more realistic assumption that the growth rate in potential output varies over time. Specifically, forward potential output growth is first calculated as $Y^*_{t,T_1,T_2} = \ln\left(\text{GDP}^*_{t+T_2}\right) - \ln\left(\text{GDP}^*_{t+T_1}\right)$, where GDP^*_t is the Congressional Budget Office potential output series as obtained from the FRED (see Congressional Budget Office (2001) for calculation details). The forward change in the output gap is then calculated as $\Delta X_{t+T_1,t+T_2} = \Delta Y_{t+T_1,t+T_2} - Y^*_{t,T_1,T_2}$.

Figure 5 plots annual GDP growth and the annual growth in potential output. Figure 6 plots the one-year forward annual change in the second measure of the output gap, and also the expected annual change in the output gap using the VAO Slope and Bow coefficients that prevailed at the time.[20] The use of annual GDP growth data is standard in this literature (e.g, see Hamilton and Kim (2002)), and so equation 15 is estimated on that basis for comparability. However, quarterly data is also used in this article to enable more precision when investigating the profile of the change in the output gap. Note that in figure 6, a material difference between the actual change in the output gap and the predicted change in the output gap based on the VAO model opens up from around the late-1970s/early-1980s. This potential structural break is discussed in section 4.1.4.

4.1.3 Measures of inflation and inflation expectations

Several measures of annual inflation and expected annual inflation are tested in the estimation of equation 17. The use of annual inflation data is standard in this literature (e.g see Kozicki (1997)).

The measures of inflation available at a quarterly frequency are CPI inflation (CPI), and inflation in the chain-type GDP deflator (IGD). The index levels for these series are obtained from the FRED, and the annual inflation rates for each quarter are calculated as $\pi_t = \ln(I_t) - \ln(I_{t-1})$, where I_t is the index level of the given price series, and $I_{t-1}$is the index level one year prior.[21] The measures of inflation expectations available at a quarterly frequency are the Survey of Professional Forecasters (SPF) expected GDP deflator for the next year (SPF IGD), and the SPF expected CPI inflation for the next year (SPF CPI). The level data for these series begins in 1968:Q4 and 1981:Q3 respectively, and are available from the Federal Reserve Bank of Cleveland website. The forecast levels are converted into inflation expectations using the trans-

[20] More specifically, the middle month of the quarter from the previous year, but the empirical results presented in this article are immaterially different using averages of the VAO coefficients for the relevant quarter, or the first or last month of the quarter.

[21] CPI inflation is all urban consumers to 1957:Q4, and ex food and energy from 1958:Q1, which is dictated by data availability.

formation $E_t(\pi_{t+1}) = \ln[E_t(I_{t+1})] - \ln[E_t(I_t)]$, where $E_t(I_{t+1})$ is the SPF forecast level one year ahead, and $E_t(I_t)$ is the SPF provisional current level that was available when the forecast was made.

The measure of inflation available on a monthly frequency is CPI inflation, as noted above. The measure of inflation expectations available on a monthly basis is the Michigan CPI inflation expectations for the next year (MIE, only available since January 1978). The data for the latter series is obtained from the FRED, and is converted to a continuously-compounding basis using the transformation $E_t(\pi_{t+1}) = \ln(1 + \text{Survey\_Rate}_t)$.

Figure 7 plots the CPI and GDP deflator and the quarterly VAO Level coefficient. Note that the difference between the measures of inflation and the VAO Level coefficient widens materially from around the late-1970s/early-1980s. This potential structural break is discussed in the following sub-section.

4.1.4 A dummy variable for structural change in the US financial and economic environment

Figures 6 and 7 both show prima facie evidence of structural breaks in the yield curve/change in output gap relationship and the yield curve/inflation relationships from around the late 1970s/early 1980s. This was not unexpected, for several reasons. Firstly, the late-1970s to the mid-1980s was a period of substantial change in the US financial and economic environment. For example, one very significant economic change was the central bank's Volcker-led disinflation, which essentially began with the change to targeting non-borrowed reserves in October 1979, and the subsequent achievement and ongoing maintenance of low inflation from the early to mid-1980s. Substantial financial deregulation also occurred from the late 1970s into the early to mid-1980s, ranging from the allowance of cheques to be written on savings account deposits, the introduction of market-interest-bearing cheque accounts, and the removal of interest rate ceilings on deposits (November 1978, 1982, and 1985 respectively; see Gordon (1990) p. 101, and pp. 503-508). There was also progressive rationalisation of reserve requirements following the Monetary Control Act of 1980 and the Depository Institutions Act of 1982. At the margin, these financial sector changes led to substitution away from Treasury securities as an investment, which would be consistent with a widening of term premia in the yield curve relative to prior history.

Secondly, there is also prior empirical evidence suggesting structural breaks to yield curve relationships within the late-1970s/early-1980s period. For example, based on statistical tests for unknown breakpoints, Estrella et al. (2003) identifies structural breaks in October 1979 and October 1982 when the yield curve is used as an indicator of future inflation, and in September 1983 when the yield curve is used as an indicator of future output. Using a similar technique, Aïssa and Jouini (2003) documents a structural break in the inflation process in June 1982. The work of Krippner (2005) also suggests a structural break in yield curve term premia from around the late-1970s/early-1980s when forecasting the yield curve out-of-sample using the VAO(3) model.

Based on this evidence, the analysis for the full sample proceeds with the

inclusion of a step dummy variable $D(t)$ in equations 15 and 17, where $D(t)$ is 0 to the period immediately before the breakpoint, and 1 from the given breakpoint. The four alternative breakpoints were tested: (1) October 1979, to coincide with the beginning of the Volcker-led disinflation; (2) October 1981, to correspond to the middle of financial and economic reform period; (3) February 1983, which is the first quarter following the introduction of Garn-St Germain Act of 1982 (signed into law on 15 October 1982); and (4) February 1984, which saw the last material decrease in the reserves held by member banks as part of the transitional phase-in of the Monetary Control Act of 1980 (see Federal Reserve Statistical Release (2004)). As it turns out, the empirical results for both the output gap and inflation were immaterially different using any of these breakpoints, and so only the results using the 1984:Q1 breakpoint, or February 1984 for monthly data, are reported.

To investigate any variation in the relationships over the full sample, the analysis is also undertaken for two sub-samples. These are pre-October 1979 and post-February 1984, which excludes all of the structural break candidates noted above.

4.2 The Slope and Bow coefficients and the change in the output gap

Table 1 contains the results from estimating equation 15 over the full sample, using the dummy variable with the 1984:Q1 breakpoint and the second measure of the change in the output gap. The results based on the first measure of the change in the output gap are similar and are so are not discussed further in the text (the tables of results are contained in Appendix 1).

The notable points from table 1 are: (1) the explanatory power of the regressions are highest for short forward horizons, and fade quickly past forward horizons of one year; (2) both the constant coefficient and the dummy coefficient are highly significant and of the expected sign for the first year, and then become insignificant but generally retain the correct sign beyond that; (3) the coefficient α_{1,T_1,T_2} (i.e the coefficient on the expected future change in the output gap based on the VAO model framework) is highly significant and positive for forward horizons up to one year, becomes insignificant while remaining positive through the second year (although the coefficient is significant on an annual basis), and becomes insignificant and negative for some forward horizons over two years; and (4) the coefficients α_{1,T_1,T_2} are insignificantly different from 1 (the value expected in theory) except for the marginal rejection of that hypothesis for the 2.25 to 2.5 year horizon.[22]

Observations 1 and 3 are consistent with prior empirical results based on annual GDP growth and 3-month/10-year spreads (e.g, see Hamilton and Kim (2002) table 2), but the VAO model framework provides some insight behind those results; i.e the decline in the explanatory power of the regressions and the significance of the coefficients occurs as the strength of the $\sum_{n=2}^{3} \beta_n(t) \cdot$

[22] The main difference using the first measure of the change in output gap (as noted in section 4.1.1) is that more of the $\Delta X_{t+T_1,t+T_2}$ coefficients were significantly different from 1 (at the 10 percent level) for forward horizons of two years or more.

$q_n(T_1, T_2)$ "signal" reduces and the ε_{T_1,T_2} "noise" increases by forward horizon. The decreasing magnitude of the constant and dummy coefficient estimates by forward horizon is also consistent with the decreasing magnitudes of $q_2(T_1, T_2)$ and $q_3(T_1, T_2)$ by forward horizon.

Most importantly, observation 4 indicates that the VAO model framework does provide a gauge of the profile (i.e the timing and magnitude) of the future change in the output gap, given that the coefficients α_{1,T_1,T_2} are not biased away from 1 for the forward horizons tested.

Table 2 contains the results for equation 15 estimated over each sub-sample, using the second measure of the output gap. The points of note are: (1) for the first sub-sample, the explanatory power of the regressions are highest for forward horizons up to one year, but fade very quickly for longer forward horizons. Conversely, for the second sub-sample, the explanatory power of the yield curve is initially low, but increases and remains more persistent for forward horizons out to two years. (2) For the first sub-sample, the coefficients are positive and highly significant for forward horizons up to one year, and become insignificant and negative for longer forward horizons. Conversely, the coefficients in the second sub-sample are positive and significant for forward horizons out to two years, and remain positive for all forward horizons tested. (3) The coefficients in the first sub-sample are initially significantly above the theoretical value of 1, while the coefficients in the second sub-sample are initially significantly below 1. In both cases, the coefficients are insignificantly different from 1 for forward horizons beyond about one year.

These sub-sample results indicate that up to 1979, the shape of the yield curve was best at predicting changes in the output gap over short forward horizons, although it tended to under-predict the magnitudes of change. Conversely, beyond 1984 the shape of the yield curve was best at predicting changes in the output gap over medium forward horizons, while remaining useful for short horizons, although it tended to over-predict the magnitudes of change. The combination of these sub-sample results evidently offset to give the coefficients close to unity on $\Delta X_{t+T_1,t+T_2}$ for the full sample.

[Table 1 here], [Table 2 here]

4.3 The Level coefficient, and inflation and inflation expectations

Table 3 contains the results of the unit root tests implied by equation 17 using the inflation data available at quarterly frequencies. A summary of the full sample results is as follows: (1) none of the inflation measures or the Level coefficient reject the null hypothesis of a unit root to usual statistical levels (except SPF CPI for the Phillips-Perron test, which seems anomalous given the augmented Dickey-Fuller test); (2) without the use of the dummy variable, only the Level less CPI series marginally rejects the unit root hypothesis; and (3) including the dummy variable, only the Level less CPI series rejects the unit root hypothesis, although the other differences become closer to significant thresholds. The sub-sample results are similar to those of the full sample; i.e CPI inflation still shows the most consistent and significant evidence of

cointegration with the VAO Level coefficient over both periods, although the cointegrating relationship with SPF CPI is marginally significant in the second sub-sample.

Table 4 contains the results of the unit root tests implied by equation 17 using the inflation data available at monthly frequencies. The results for CPI inflation and the VAO Level coefficients are very similar to those already noted using a quarterly frequency. The results for MIE (only available in the second sub-sample) indicate that this measure of inflation expectations is not a unit root series. This might be attributable to some sharp falls and subsequent reversals in that series (e.g following the 11 September 2001 World Trade Centre tragedy). The Level less MIE series marginally rejects the unit root hypothesis.

Overall, the results confirm the predicted homogeneous cointegration relationship between the VAO Level and CPI inflation. The results for alternative measures of inflation and/or measures of inflation expectations are certainly not as convincing, but generally show weak evidence of homogeneous cointegration relationships with the VAO Level. The reason for these results is open to speculation, but one hypothesis is that the market considers CPI inflation to be the most relevant measure of inflation, perhaps because the central bank has tended to focus most on measures of inflation that are based on consumer prices.

5 Conclusion and implications for monetary policy

The volatility-adjusted orthonormalised Laguerre (VAO) model of the yield curve offers a new and straightforward framework for investigating and interpreting the relationships between the yield curve, output, and inflation. This article shows theoretically and confirms empirically (using US data) that the level of the yield curve as measured by the VAO model has a cointegrating relationship with inflation, and the shape of the yield curve as measured by the VAO model corresponds to the profile (i.e timing and magnitude) of future changes in the output gap.

The implication for monetary policy is that the central bank should be able to use the VAO model of the yield curve to gauge the market's central expectation on the key variables of inflation and the output gap (i.e by adding predicted changes in the output gap to an estimate of the current output gap). The central bank can compare those expectations against it's own policy targets, preferences, and economic forecasts, and potentially adjust policy, the communication of the goals of policy, and/or its economic forecasts as appropriate.

As an example, an inflation-targeting central bank would typically prefer the VAO Level coefficient to remain reasonably steady over time, which is consistent with well-anchored inflation expectations (assuming no material changes to estimates of growth in potential output). Assuming output and interest rate volatility are also given some weight in the central bank's preferences, the central bank would also prefer the shape of the yield curve as summarised by the non-Level VAO coefficients to be consistent with a "smooth" transition back

to a neutral output gap; anything different would potentially lead to an undershoot or overshoot of the neutral output gap, which might then threaten the inflation target and/or necessitate more aggressive policy movements in the future.[23]

A The yield curve/output results based on the first measure of the change in the output gap

[Table 5 here], [Table 6 here]

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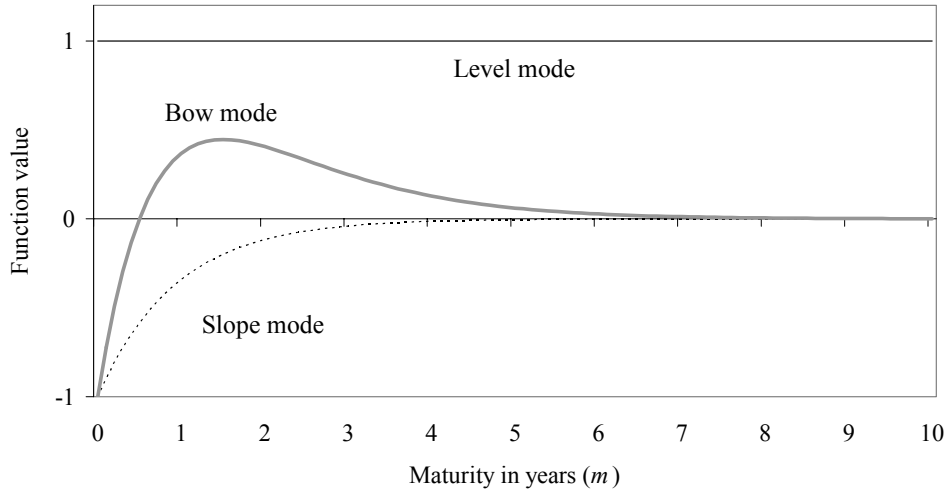

Figure 1: An illustration of the Level, Slope, and Bow modes (using $\phi = 1$) that are used to represent the expected path of the short rate in the VAO model.

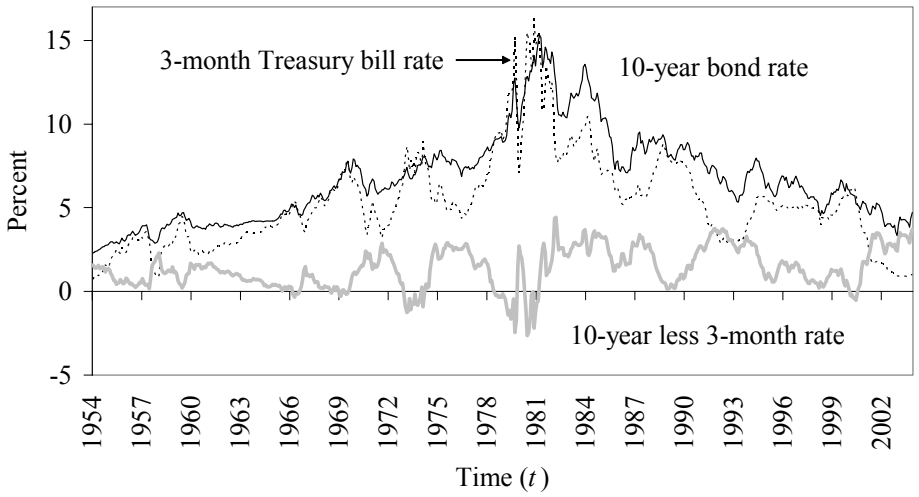

Figure 2: The time-series of two of the seven interest rates series used to estimate the VAO model coefficients. The 3-month/10-year spread is not used in the analysis, and is only shown for comparison to the Slope coefficient series in figure 4.

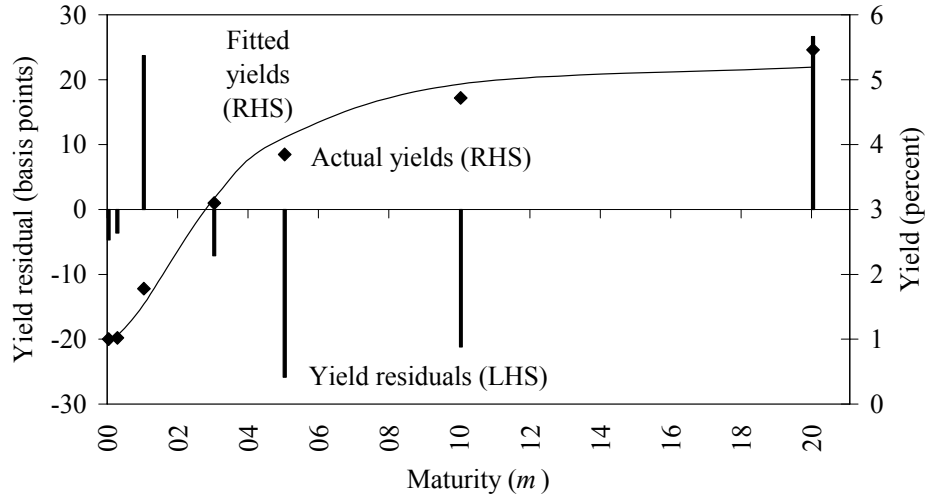

Figure 3: The cross-sectional yield curve data for May 2004, and the estimated yields using the VAO(3) model. The estimated Level, Slope, and Bow coefficients are, respectively, β_1 (May-04) = 6.00%, β_3 (May-04) = 7.58%, and β_3 (May-04) = −2.64%.

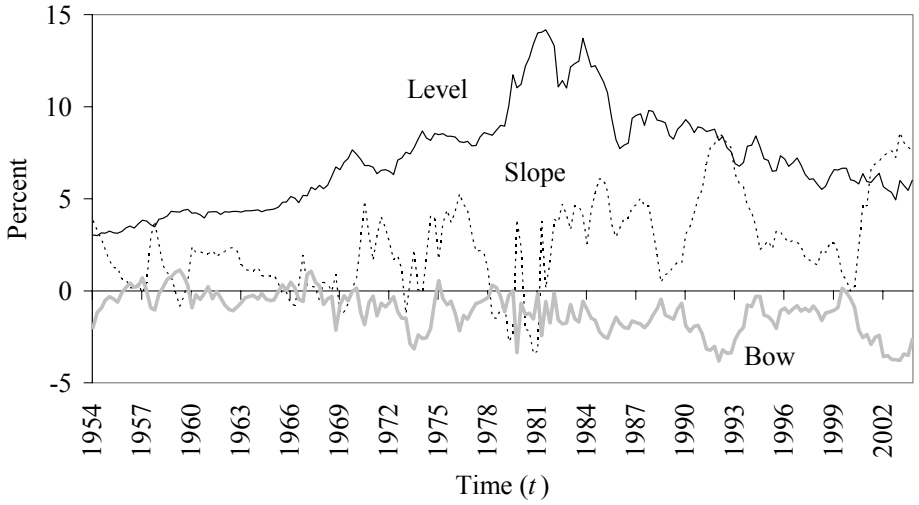

Figure 4: The time series of the estimated VAO model Level, Slope, and Bow coefficients (i.e $\beta_1(t)$, $\beta_2(t)$, and $\beta_3(t)$). The coefficients at each point in time are estimated using the seven points of yield curve data observed at that point in time.

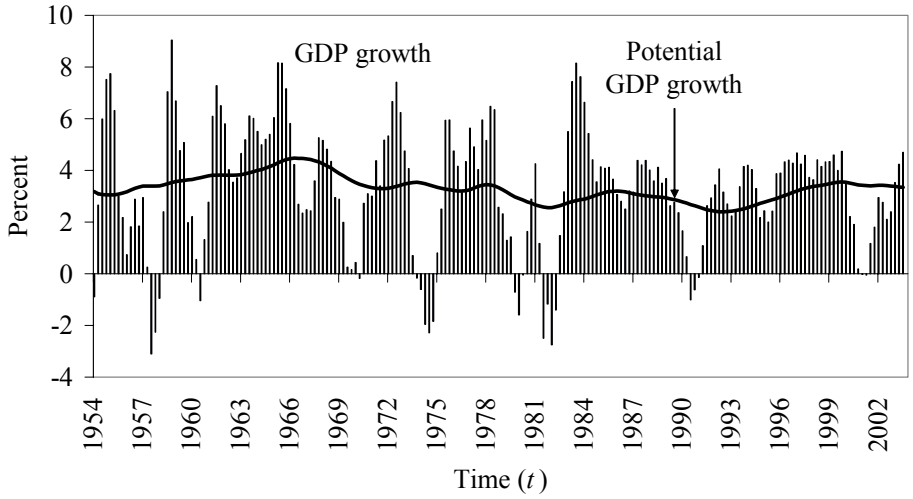

Figure 5: Annual growth in gross domestic product (GDP), and annual growth in the Congressional Budget Office estimate of potential GDP. The difference between these series is the measure of the annual change in the output gap in figure 6.

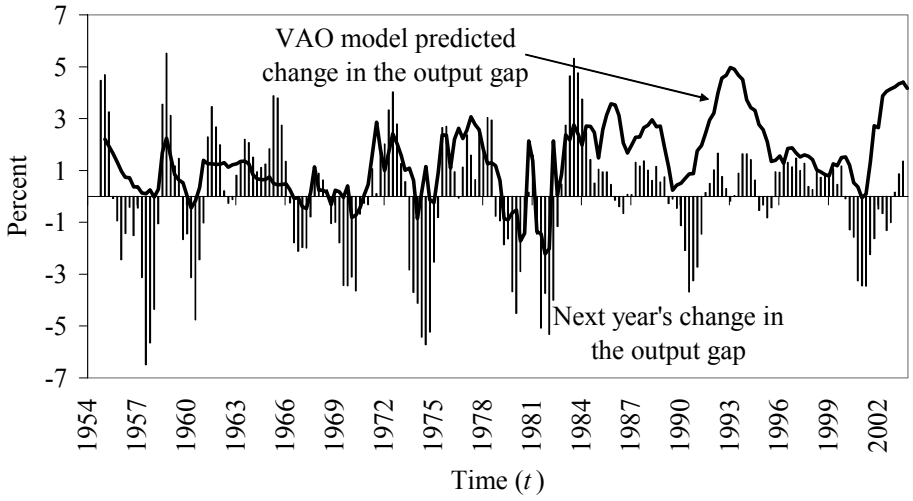

Figure 6: The time series of the change in the output gap (calculated from the data plotted in figure 5), and the predicted change in the output gap based on the VAO model framework. Note the apparent level shift in the relationship from around the late-1970s/early-1980s, as discussed in section 4.1.4.

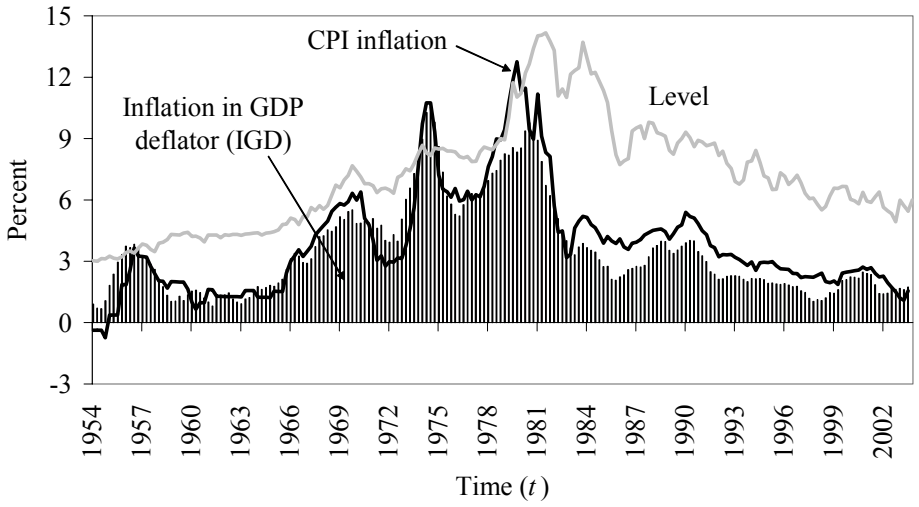

Figure 7: The time series of the VAO model Level coefficient and two measures of inflation. Note the apparent widening of the spread between the Level coefficient and the measures of inflation from around the late-1970s/early-1980s, as discussed in section 4.1.4.

Horizon	$q_n(T_1,T_2)$		R-sq.	Constant	$\alpha_{I,}\,T_1,T_2$	Dummy	$\alpha_{I,}\,T_1,T_2$
T_1 , T_2	Slope	Bow		coefficient	coefficient	coefficient	less 1
0 - 0.25	0.88	0.65	12.7	-0.76 **	0.88 ***	-1.23 **	-0.12
0.25 - 0.5	0.67	0.14	18.2	-1.09 **	1.21 ***	-1.83 ***	0.21
0.5 - 0.75	0.51	-0.17	11.8	-1.02 **	1.09 ***	-1.47 **	0.09
0.75 - 1	0.39	-0.34	7.0	-0.88 *	0.95 ***	-1.10 *	-0.05
1 - 1.25	0.30	-0.42	5.4	-0.82	0.96 ***	-0.94	-0.04
1.25 - 1.5	0.23	-0.44	1.6	-0.51	0.59	-0.41	-0.41
1.5 - 1.75	0.18	-0.43	0.6	-0.31	0.44	-0.29	-0.56
1.75 - 2	0.13	-0.40	0.5	-0.29	0.46	-0.24	-0.54
2 - 2.25	0.10	-0.36	0.1	-0.15	0.26	-0.12	-0.74
2.25 - 2.5	0.08	-0.32	0.1	-0.03	-0.17	0.19	-1.17 *
2.5 - 2.75	0.06	-0.28	0.0	-0.05	-0.08	0.11	-1.08
2.75 - 3	0.05	-0.24	0.2	-0.18	0.64	-0.14	-0.36
0 - 1	0.61	0.07	30.8	-0.96 ***	1.06 ***	-1.43 ***	0.06
1 - 2	0.21	-0.43	3.7	-0.47	0.63 ***	-0.46	-0.37
2 - 3	0.07	-0.30	0.0	-0.09	0.10	0.00	-0.90

Table 1: Full-sample estimates of equation 15 using the dummy variable with the 1984:Q1 breakpoint, and the second measure of the change in the output gap. ***, **, * respectively represent 1, 5, and 10 percent two-tailed levels of significance.

	Up to 1979:Q3			From 1984:Q1		
Horizon		$\alpha_{I,}{}_{T_1,T_2}$	$\alpha_{I,}{}_{T_1,T_2}$		$\alpha_{I,}{}_{T_1,T_2}$	$\alpha_{I,}{}_{T_1,T_2}$
T_1 , T_2	R-squared	coefficient	less 1	R-squared	coefficient	less 1
0 - 0.25	20.6	1.59 ***	0.59 *	2.6	0.31 **	-0.69 ***
0.25 - 0.5	17.4	1.79 ***	0.79 **	4.0	0.38 *	-0.62 ***
0.5 - 0.75	12.5	1.72 ***	0.72 *	6.4	0.50 **	-0.50 **
0.75 - 1	8.8	1.62 ***	0.62	4.5	0.50 *	-0.50 *
1 - 1.25	3.2	1.08 *	0.08	3.1	0.50 *	-0.50 *
1.25 - 1.5	0.2	0.32	-0.68	3.8	0.65 **	-0.35
1.5 - 1.75	0.2	0.33	-0.67	3.8	0.77 **	-0.23
1.75 - 2	0.0	-0.14	-1.14	2.5	0.79 *	-0.21
2 - 2.25	0.0	-0.21	-1.21	1.7	0.83 *	-0.17
2.25 - 2.5	0.8	-1.10	-2.10	0.1	0.49	-0.51
2.5 - 2.75	0.9	-1.38	-2.38 *	0.0	0.43	-0.57
2.75 - 3	0.1	0.52	-0.48	0.0	0.41	-0.59
0 - 1	39.6	1.80 ***	0.80 ***	14.6	0.39 **	-0.61 ***
1 - 2	1.0	0.43	-0.57	13.6	0.65 **	-0.35
2 - 3	0.4	-0.49	-1.49	2.1	0.52	-0.48

Table 2: Sub-sample estimates of equation 15 using the second measure of the change in the output gap. ***, **, * respectively represent 1, 5, and 10 percent two-tailed levels of significance.

	Full sample		Full sample with 1984:Q1 dummy		Up to 1979:Q3		From 1984:Q1	
	PP	ADF	PP	ADF	PP	ADF	PP	ADF
IGD	-1.95	-1.47			-1.07	-0.66	-1.88	-1.12
SPF IGD	-1.23	-0.93			-0.31	-0.28	-1.73	-0.98
CPI	-2.38	-1.28			-1.18	-0.35	-1.04	-0.62
SPF CPI	n/a	n/a			n/a	n/a	-1.33	-1.42
Level	-1.72	-1.66			-0.18	-0.37	-2.26	-2.25
- IGD	-1.94	-1.71	-2.67	-2.35	-2.49	-3.13 **	-2.13	-2.24
- S_IGD	-2.20	-2.11	-2.38	-2.22	-0.96	-1.28	-2.49	-2.61 *
- CPI	-2.70 *	-2.68 *	-3.47 **	-3.88 **	-2.62 *	-3.18 **	-2.70 *	-2.84 *
- S_CPI	-1.67	-1.87	-2.53	-2.42	n/a	n/a	-2.66 *	-2.68 *

Table 3: Phillips-Perron (PP) and augmented Dickey-Fuller (ADF) unit root tests on quarterly frequency data. IGD is annual inflation in the GDP deflator, SPF IGD is expected year-ahead IGD, CPI is annual CPI inflation, SPF CPI is expected year-ahead CPI, Level is the VAO model Level coefficient. The bottom part of the table are tests for Level less the given inflation measures. The left-hand side results are for the full sample and the full sample with a 1984:Q1 structural break. The right-hand side of the table contains the sub-sample results. "n/a" indicates no or insufficient data for the test. ***, **, * respectively represent 1, 5, and 10 percent levels of significance.

	Full sample		Full sample with Feb-1984 dummy		Up to Sep-1979		From Feb-1984	
	PP	ADF	PP	ADF	PP	ADF	PP	ADF
CPI	-2.33	-1.39			-1.19	-1.16	-1.22	-1.02
MIE	n/a	n/a			n/a	n/a	-4.54 ***	-4.63 ***
Level	-1.81	-1.86			-0.25	-0.16	-2.48	-3.25 **
- CPI	-2.75 *	-2.46	-3.45 **	-3.48 **	-2.78 *	-3.29 **	-3.14 **	-4.17 ***
- MIE	n/a	n/a	n/a	n/a	n/a	n/a	-2.57 *	-2.62 *

Table 4: Phillips-Perron (PP) and augmented Dickey-Fuller (ADF) unit root tests on monthly frequency data. CPI is annual CPI inflation, MIE is Michigan CPI inflation expectations for the year ahead, Level is the VAO model Level coefficient. The bottom part of the table are tests for Level less the given inflation measure. The left-hand side results are for the full sample, and the full sample with a February 1984 structural break. The right-hand side of the table contains the sub-sample results. "n/a" indicates no or insufficient data for the test. ***, **, * respectively represent 1, 5, and 10 percent levels of significance.

Horizon	$q_n(T_1,T_2)$		R-sq.	Constant	$\alpha_{I,}\,T_1,T_2$	Dummy	$\alpha_{I,}\,T_1,T_2$
T_1 , T_2	Slope	Bow		coefficient	coefficient	coefficient	less 1
0 - 0.25	0.88	0.65	11.4	-0.57	0.84 ***	-1.62 ***	-0.16
0.25 - 0.5	0.67	0.14	16.2	-0.87 *	1.15 ***	-2.18 ***	0.15
0.5 - 0.75	0.51	-0.17	9.9	-0.79	1.01 ***	-1.80 ***	0.01
0.75 - 1	0.39	-0.34	5.5	-0.64	0.85 ***	-1.41 **	-0.15
1 - 1.25	0.30	-0.42	4.0	-0.57	0.83 ***	-1.23 *	-0.17
1.25 - 1.5	0.23	-0.44	0.9	-0.25	0.43	-0.70	-0.57
1.5 - 1.75	0.18	-0.43	0.4	-0.05	0.25	-0.58	-0.75 *
1.75 - 2	0.13	-0.40	0.4	-0.03	0.24	-0.54	-0.76 *
2 - 2.25	0.10	-0.36	0.3	0.11	0.00	-0.42	-1.00 *
2.25 - 2.5	0.08	-0.32	0.5	0.22	-0.48	-0.12	-1.48 **
2.5 - 2.75	0.06	-0.28	0.4	0.20	-0.43	-0.20	-1.43 *
2.75 - 3	0.05	-0.24	0.3	0.06	0.22	-0.46	-0.78
0 - 1	0.61	0.07	25.6	-0.74 *	0.99 ***	-1.78 ***	-0.01
1 - 2	0.21	-0.43	2.3	-0.21	0.45 *	-0.75	-0.55 **
2 - 3	0.07	-0.30	0.8	0.16	-0.23	-0.31	-1.23 **

Table 5: Full-sample estimates of equation 15 using the dummy variable with the 1984:Q1 breakpoint, and the first measure of the change in the output gap. ***, **, * respectively represent 1, 5, and 10 percent two-tailed levels of significance.

	Up to 1979:Q3			From 1984:Q1		
Horizon		$\alpha_{I,}{}_{T_1,T_2}$	$\alpha_{I,}{}_{T_1,T_2}$		$\alpha_{I,}{}_{T_1,T_2}$	$\alpha_{I,}{}_{T_1,T_2}$
T_1 , T_2	R-squared	coefficient	less 1	R-squared	coefficient	less 1
0 - 0.25	17.9	1.49 ***	0.49	2.6	0.23	-0.77 ***
0.25 - 0.5	14.5	1.65 ***	0.65 *	4.0	0.29	-0.71 ***
0.5 - 0.75	9.8	1.54 ***	0.54	6.4	0.41	-0.59 **
0.75 - 1	6.5	1.41 ***	0.41	4.5	0.38	-0.62 **
1 - 1.25	2.0	0.85	-0.15	3.1	0.37	-0.63 **
1.25 - 1.5	0.0	0.08	-0.92	3.8	0.48	-0.52
1.5 - 1.75	0.0	0.06	-0.94	3.8	0.57	-0.43
1.75 - 2	0.2	-0.43	-1.43 *	2.5	0.55	-0.45
2 - 2.25	0.3	-0.53	-1.53	1.7	0.55	-0.45
2.25 - 2.5	1.4	-1.46	-2.46	0.1	0.16	-0.84
2.5 - 2.75	1.5	-1.77	-2.77 **	0.0	0.05	-0.95 *
2.75 - 3	0.0	0.13	-0.87	0.0	-0.02	-1.02
0 - 1	30.7	1.60 ***	0.60 *	7.7	0.29	-0.71 ***
1 - 2	0.0	0.03	-0.97 *	6.3	0.45	-0.55
2 - 3	1.2	-0.90	-1.90	0.2	0.15	-0.85 **

Table 6: Sub-sample estimates of equation 15 using the first measure of the change in the output gap. ***, **, * respectively represent 1, 5, and 10 percent two-tailed levels of significance.