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Estimating the Final Size of an Online User Base

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Abstract

The theoretical insights from the increasing returns literature, plus the interaction between consumers facilitated by networked technologies, have led to a synthesis in which virtual communities become uniquely valuable to an online firm. Strategy in social media markets, in particular, becomes one of promoting information sharing and connectivity within networks of user communities, deepening the relationship between the user base and sellers, and paving the way for a revenue payoff. When network externalities also suggest the possibility of barriers to entry and lock-in operating on the demand side, the importance of a large user base correspondingly increases. From a finance perspective the relevant question then is: how large will a firm’s user base eventually become? Cauwels and Sornette (2011) answer this question by positing an S-shaped model of user growth. We extend their model by introducing competition from another online firm. With this extension, S-shaped growth is altered, potentially invalidating Cauwels and Sornette’s (2011) results.

Keywords

user base growth
Facebook valuation
S-curves

JEL Codes

C15, D85, G17

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1. Introduction

Capital continues to flock to high-profile social media companies, with IPO valuations well in excess of those implied by current profits. To traditional economists the rush for Internet companies to go public, and the willingness of investors to back them, creates a number of puzzles. If a company isn't making much profit today, why would it be making a profit tomorrow when more competitors have entered the market? Why has the average Internet start-up gone public in a fraction of the time that traditional companies have taken, despite the greater uncertainty that a quick IPO entails? Why have metrics such as the user base replaced traditional measures of market valuation (Perkins and Perkins 1999:62, Ashwin, Daley and Taylor 2000) and been instrumental in attracting venture and other forms of capital?

In attempting to provide a partial answer to the above puzzles, we suggest that the puzzles themselves and increasing returns to scale are linked. Rushing into a market makes sense if it allows a firm to lock in customers and lock out rivals. Self-reinforcing networks of online users can be pivotal in establishing entry barriers (Bates *et al*. 2001). The Internet becomes a way to increase market dominance through increasing returns to providing information and sharing experiences between users. If information and connectivity are the only factors differentiating a product across firms, then the website that provides more information and connectivity boosts its customer base. Developing a lead in building a user community, and therefore a lead in connecting users or gaining information from them, makes a website more attractive, further increasing the user base and ultimately its IPO share price.

It follows that first mover advantages may be crucial in online markets. Given the increasing returns enjoyed by a first mover, any subsequent mover faces difficulties in persuading the incumbent's user community to switch. The follower falls behind as its smaller online community supplies relatively less information or connectivity. Over the last decade virtual user communities have been recognised as distinct e-commerce business models (Bughin and Hagel 2000:237, Afuah and Tucci 2001:47). This has led some commentators to suggest that competition in electronic markets becomes a race to build a user community (for example, Evans and Wurster 2000:100; Hagel and Armstrong 1997:2). Building a user community, however, is not the same as building web traffic. Web traffic metrics were popular in valuing dot.coms during the Internet boom of the last century. But the lavish expenditure on attracting visitors to websites largely failed to impact significantly on visitor growth, retention, or conversion of visits into spending. Given the abrupt downturn in dot.coms a dozen or so years ago, post-crash metrics of e-performance have begun to focus more tightly on the user communities themselves (for example, Agrawal, Arjona and Lemmens 2001).

We present the microeconomic foundations of the S-curve (logistic) growth of an online company's user base. This work extends the important research undertaken by Cauwels and

Sornette (2011) in valuing Facebook prior to its IPO. These authors make the valid point that if the asymptotic growth plateau implied by a logistic equation is not taken into account, the valuation of a company at its IPO could be significantly overestimated. The growth of Facebook's user base cannot be exponential indefinitely – there must be an upper limit to it, described as the 'carrying capacity' or the maximum number of potential users. But the focus on measuring the 'carrying capacity', and representing this as the actual or equilibrium number of users to used for valuation purposes, may be misleading. The method proposed by Cauwels and Sornette (2011) may also overestimate the final size of a user base, at least in the case where the 'carrying capacity' and the equilibrium diverge. Cauwel and Sornette's problem is their implicit assumption that there is only a single firm in the industry. In this case, the 'carrying capacity' and the non-trivial equilibrium user base always coincide. In contrast, we posit competition from another firm, where the growth of the user community of one firm may pull users away from the other. This approach seems more in line with the actual market situation facing online companies. With this extension we provide a model that we believe will yield more accurate econometric predictions of the ultimate size of a user base, an estimate that can be used for IPO valuations. We show the conditions under which Cauwels and Sornette's measure of the 'carrying capacity' coincides with the equilibrium user base, and when it does not. Our results deepen the understanding of user base metrics in valuing an online, social media company.

2. User Communities and Increasing Returns

Traditional methods of informing consumers have typically encountered a trade-off between the quality of information (richness) and the number of people receiving it (reach). In the past, delivering richer information may have required close physical proximity between the communicator and receiver, or required a dedicated channel such as a sales force. This narrowed the reach of the information. But electronic networks have overturned the richness-reach trade-off. Large numbers of people, connected digitally, are now able to exchange more and higher quality information almost instantaneously at very low cost. With the spread of e-commerce and different forms of uncertainty for consumers, imparting information has the potential to become an important source of competitive advantage for online firms. eDiets, for instance, purported to be the most popular diet and exercise site on the Web, assigns a successful program user to mentor each new member. The members of eDiets' virtual community give each other motivational boosts, and members are encouraged to use the chat rooms, bulletin boards and online seminars (Taylor 2001:61).

A virtual community has been described as being more important to a firm than the type or amount of resources that it owns (Hagel and Armstrong 1997:14). Whether the communities are built around products or socio-demographic categories, the interaction between community members can reinforce community retention and growth. In contrast to straight web traffic, online user communities offer greater advantages in a world where rivals' websites are just an instant away. According to Agrawal, Arjona and Lemmens

(2001), less than one percent of all visits to a transaction site come from people who become repeat customers. Yet online community sites have a 60 percent success rate in converting repeat visitors into members/customers, with an average member retention rate of up to 18 percent. In contrast to transactions sites, user community sites tend to maintain their conversion rate of visitors into repeat customers as visitor traffic rises, which should allow them to build stronger franchises than transactions sites of the same size. Even though consumers are able to switch between firms and become a user of either, often users choose not to. Marn (2000) suggests that for only around eight percent of active online consumers is price the major factor influencing purchase behavior, with more than 90 percent of CD buyers and 80 percent of book shoppers visiting only a single site, despite prices between online sellers varying on average by at least 25 percent.

3. The Chatterjee and Eliashberg Model

We begin by linking network effects with diffusion models, an approach that is gaining prominence in the business literature (see, for example, Valente 1995). Diffusion models themselves have been used extensively in the marketing and management science areas (eg, Mahajan, Muller and Bass 1990, Baptista 1999) and in theoretical biology, whose general ideas form the basis of some of the following sections of the paper.

The modeling initially follows Chatterjee and Eliashberg (1990), who provide a microeconomic foundation for an aggregate pattern of sales growth of a new product or service (an innovation). In their model individuals evaluate an innovation based on price and performance. For our purposes, let the innovation be membership of a website's user community. Membership includes active participation or interaction with others in the user community. Members are likely to become customers of the website that they join. Potential members know the price of membership, but are uncertain about the performance or services obtained from membership of the user community. Perceptions of performance may change over time as individuals obtain more information or feedback from the user community, particularly about the product that the website sells.

Suppose a potential member's risk aversion is given by the following utility function:

$$u_x(x_i) = 1 - \exp(-cx_i), \qquad (1)$$

where x_i, a random variable, is the individual's uncertain perception of the user community's performance after receiving i units of information from the community about the product sold on the website. c (>0) is a coefficient of risk aversion.

Let the individual's utility function for membership of the user community be:

$$u(x_i, p) = k_x u_x(x_i) + k_p u_p(p), \qquad (2)$$

where p is the price of membership and k_x and k_p are the weights associated with the respective performance and price utility functions. Assuming that the utility for price is linear in its argument, (2) can be rewritten as:

$$u(x_i, p) = u_x(x_i) + ku_p(p) = 1 - \exp(-cx_i) - kp, \qquad (3)$$

where k represents the relative importance of price. With the arguments in the utility function scaled relative to status quo and $u(0,0) = 0$, the utility from the status quo (not joining) is zero. The individual joins the user community if:

$$E[u_x(x_i)] > kp. \qquad (4)$$

If the random variable x_i is normally distributed with mean m_i (indicting the individual's expectation of performance) and variance s_i^2 (indicating perceptual uncertainty), then:

$$E[u_x(x_i)] = E[1 - \exp(-cx_i)] = 1 - \exp(-cm_i + c^2 s_i^2 / 2). \qquad (5)$$

Given (4) and (5) the condition for membership is:

$$m_i > cs_i^2 / 2 - (1/c)\ln(1 - kp). \qquad (6)$$

Restricting the analysis to $kp<1$, the individual becomes a member of the user community once the expectation of the community's performance (m_i) exceeds the sum of the price barrier [$-(1/c)\ln(1-kp)$] and the risk barrier ($cs_i^2/2$).

The dynamics of the individual's perception depend on the initial perception at the time of website launch and the type of information flow. The individual receives a stream of information about the website's product from the website's user community. Let the potential consumer behave as if the performance level associated with a unit of information is sampled from a normal distribution with mean μ and variance σ^2. μ represents the true performance of the user community relative to the status quo and is unknown to the individual.

Chatterjee and Eliashberg (1990) make the following definitions:

$$y_i = [cs_i^2 / 2 - (1/c)\ln(1 - kp) - m_i][\sigma^2 / s_i^2] \text{ and} \qquad (7)$$

$$\chi = y_0 = [cs_0^2 / 2 - (1/c)\ln(1 - kp) - m_0][\sigma^2 / s_0^2] \qquad (8)$$

$$\lambda = -(1/c)\ln(1 - kp). \qquad (9)$$

In relation to (6), the individual becomes a member of the user community if:

$$y_i < 0, \tag{10}$$

while people with χ's less than zero become members as soon as the website is available.

In the model so far the dynamics of membership have been expressed in discrete units of information. In moving to a continuous time model, i becomes a continuous variable representing the cumulative amount of information. Using $y(i)$ to denote the continuous process the individual, as before, becomes a member once $y(i)$ becomes strictly negative. If an individual is characterized by $\mu > \lambda$, he or she drifts towards membership since the true performance exceeds the price barrier.

A person for whom $\chi \geq 0$ will not become a member immediately after the launch of the website. Purchase will require a critical, cumulative amount of information. Let the critical amount of information be a random variable, $i^* | \chi, \lambda$, having an inverse Gaussian distribution with mean:

$$E[i^* | \chi, \lambda] \equiv \tau . \tag{11}$$

Note that:

$$i(t) = \int_0^t n(t)dt, \tag{12}$$

where $n(t)$ is the rate of information and t=0 is when the website is launched.

To derive an aggregate model of innovation diffusion, Chatterjee and Eliashberg (1990) aggregate across individuals based on the mean of the distribution $i^* | \chi,\lambda$ (rather than across the probability distribution of $i^* | \chi,\lambda$). In their approach some consumers adopt the innovation upon launch (Type I consumers), others adopt after receiving the critical amount of information, $i^* | \chi,\lambda = \tau$ (Type II), and rest do not adopt at all.

Denote ψ_I and ψ_{II} as the population shares of Type I and II individuals. If the distribution of τ across Type II individuals is given by the density function $f_\tau(\cdot)$ and the cumulative distribution function by $F_\tau(\cdot)$, the cumulative penetration of membership is:

$$A(t) = \psi_I + \psi_{II} F_\tau (i(t)), \qquad 0 \leq A(t) \leq 1. \tag{13}$$

The penetration rate is:

$$\dot{A}(t) = \psi_{II} n(t) f_{\tau}(i(t)), \qquad (14)$$

where $f_\tau(\cdot)$ is the concentration of people who are "ready" to become members of the user community.

Suppose:

$$n(t) = n_0 A(t); \qquad (15)$$

that is, the information rate increases linearly with cumulative penetration. This could be the impact of reach in stimulating information flows from an online user community. Let this increase the benefit of joining the website's user community.

Suppose further that $\psi_I = A_0$, $\psi_{II} = 1 - A_0$ and $0 < A_0 < 1$. From (14):

$$\dot{A}(t) = n_0(1 - A_0)A(t)[1 - A(t)], \qquad (16)$$

where the density $f_\tau(i(t))$, representing people who are "ready" to become members, has been replaced by $[1\text{-}A(t)]$, assuming that people who are yet to be members are identical and are equally likely to become members.

We convert the penetration rate into a user community rate by multiplying the penetration rate, a fraction, by the website's fixed population of potential members. A now represents the number of members in the user community. Let $\overline{A}$ represent the fixed limit to the size of the user community, that is, the 'carrying capacity' in Cauwels and Sornette (2011). The limit will affect the growth rate of a website (or firm's) user community; suppose the growth rate of the user community falls as the actual user community tends towards its limit. Denoting $\gamma = n_0(1 - A_0)$ as the intrinsic growth rate of the user community, and deleting the t's for notational expedience, we have:

$$\dot{A} = \gamma\left(1 - \frac{A}{\overline{A}}\right)A \qquad (17)$$

Equation (17) models logistic growth of the user community. The growth rate of A falls with increases in A. For example, if $A = \overline{A}$ the logistic growth rate of A is zero. It is straightforward to show that as $t \to \infty$, $A \to \overline{A}$.

4. Extending the Model

We modify (17) to model a race for user community membership between two firms, A and B. Let competition by one firm impact adversely on the user community of the other. For example, suppose that of the consumers searching for a product, some locate Firm B's website first. Let a proportion of them, *c*, stay with Firm B. This assumption conforms to empirical studies. Some websites lock in first-time users better than their rivals do, or, after spending time to complete information requested by one site, users may be reluctant to pursue other sites (Ashwin, Daley and Taylor 2000:23). The existence of Firm B reduces Firm A's growth rate. If *A*(*t*) and *B*(*t*) represent the total membership of the user communities of each firm, then for Firm A, say:

$$\begin{aligned}\dot{A} &= \gamma\left(1-\frac{A+B}{\overline{A}}\right)A \\ &= \gamma\left(1-\frac{A}{\overline{A}}-cB\right)A, \quad c = 1/\overline{A} \\ &= \left(\gamma-\gamma\frac{A}{\overline{A}}-\beta B\right)A, \quad \beta = \gamma c.\end{aligned}$$

We have a system of two competing firms, A and B, whose user community growth rates are given by:

$$\dot{A} = \left(\gamma_A-\gamma_A\frac{A}{\overline{A}}-\beta B\right)A, \text{ and} \tag{18}$$
$$\dot{B} = \left(\gamma_B-\gamma_B\frac{B}{\overline{B}}-\alpha A\right)B.$$

γ_A and γ_B are the intrinsic growth rates of the communities. Let γ_A, γ_B, α and β be strictly positive constants. *A* and *B* have upper limits to the user/member populations denoted by $\overline{A}$ and $\overline{B}$. The coefficients α and β represent the inhibiting effects of *A* on *B* and *B* on *A*. Several assumptions have been made in the model. In attracting members, firms do not compete on the basis of price. And if both *A* and *B* are very small, both are able to increase; ie., the bracketed terms in (18) are strictly positive.

In general the system (18) cannot be solved explicitly. However, the fixed points can be determined and the paths of *A* and *B* described in phase diagrams. There are four (*A*,*B*) fixed points:

$(0,0)$, $(0,\overline{B})$, $(\overline{A},0)$, (a,b), where $a = (\frac{1}{\overline{B}} - \frac{\beta}{\gamma_A}) / (\frac{1}{\overline{A}\overline{B}} - \frac{\alpha\beta}{\gamma_A \gamma_B})$, and

$$b = (\frac{1}{\overline{A}} - \frac{\alpha}{\gamma_B}) / (\frac{1}{\overline{A}\overline{B}} - \frac{\alpha\beta}{\gamma_A \gamma_B}) . \qquad (19)$$

We will use phase diagrams to analyse scenarios relating to the approach used by Cauwels and Sornette (2011), using with a model of two firms.

5. Two Competing Firms

Assume two firms, Firm *A* and Firm *B*. Firm *B* will be the firm whose user plateau or carrying capacity we will explore. Let the firms be myopic, in the sense that they fail to take into account strategic reactions by their competitor. We shall see that the final size of a firm's user base depends on the characteristics of a firm's market segment. Ultimately we will show the conditions under which the 'carrying capacity' and final size of a user base converge or diverge.

At $t(0)$, before any firm enters the market, we make the following assumption:

ASSUMPTION 1 (Identical firms): $\overline{A} = \overline{B}$, $\gamma_A = \gamma_B$, and $\alpha = \beta$.

The following phase diagrams illustrate two interpretations of Assumption 1. Lines are drawn for $\dot{A} = \dot{B} = 0$, with the arrows indicating the path orientation of *A* and *B* in the regions bounded by the isoclines.

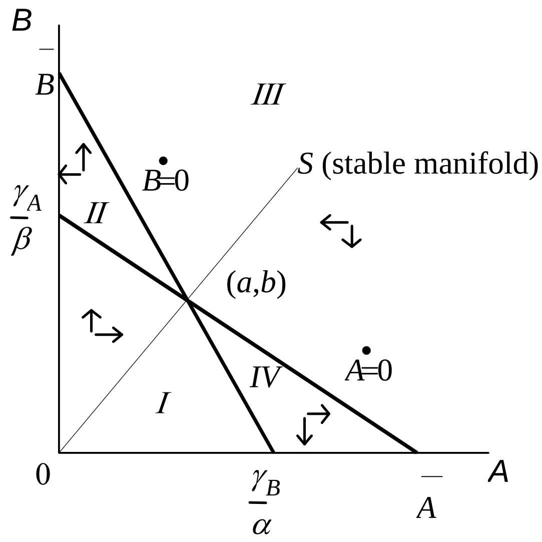
Case 1: $\overline{A} = \overline{B} > \gamma_B / \alpha = \gamma_A / \beta$

In Figure 1 there is an interior fixed point (a saddle), the origin is a source and the two fixed points on the boundary are sinks.

LEMMA 1: *If* $\overline{A} = \overline{B} > \gamma_B / \alpha = \gamma_A / \beta$*, there exists a stable manifold, denoted S, through (a,b). The stable manifold contains the origin.*

Proof: Omitted.

Figure 1. The ‘Carrying Capacity’ and Equilibrium User Base Converge



CASE 1

The first mover dominates the market as time becomes sufficiently large. Pre-emption of the competition is achieved and the ‘carrying capacity’ equals the equilibrium user base as time becomes large:

ASSUMPTION 2 (Firm A is the first mover): *Let* $(A(0), B(0))=(A_1, 0), 0<A_1<\gamma_B/\alpha$.

PROPOSITION 1 *Let Lemma 1 and Assumption 2 hold. Then A and B rise initially;* $\lim_{t\to\infty}(A(t), B(t)) = (\bar{A},0)$.

Proof: Proposition 1 follows from the construction of the phase diagram. From Lemma 1, since both firms are identical, a stable manifold of 45^o extends from the origin to (a, b). From Assumption 2, the initial high value of A relative to B generates a trajectory lying to the right of the 45^o line. On this trajectory B can rise, but the rise cannot be sustained over time. The trajectory eventually leads into region IV and fixed point $(\bar{A},0)$. □

PROPOSITION 2 The fixed point $(\bar{A},0)$ is asymptotically stable.

Proof: Let $A = A^* + c$ and $B = B^* + s$, where c and s are small. Linearize the system of (18) in the neighborhood of the fixed point, taking the first two terms of a Taylor series:

$$\dot{c} = a_1 c + a_2 s,$$
$$\dot{s} = b_1 c + b_2 s. \qquad (20)$$

The coefficients are the respective partial derivatives evaluated at the fixed point:

$$a_1 = -\gamma_A, \quad a_2 = \beta\overline{A}, \quad b_1 = 0, \quad b_2 = \gamma_B - \alpha\overline{A}.$$

$$\therefore \begin{pmatrix} \dot{c} \\ \dot{s} \end{pmatrix} = \begin{pmatrix} -\gamma_A & \beta\overline{A} \\ 0 & \gamma_B - \alpha\overline{A} \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix}. \tag{21}$$

It is necessary to find eigenvalues ξ satisfying:

$$\det\begin{pmatrix} -\gamma_A - \xi & \beta\overline{A} \\ 0 & \gamma_B - \alpha\overline{A} - \xi \end{pmatrix} = 0$$

$$\Rightarrow \xi = -\gamma_A, \quad \xi = \gamma_B - \alpha\overline{A}. \tag{22}$$

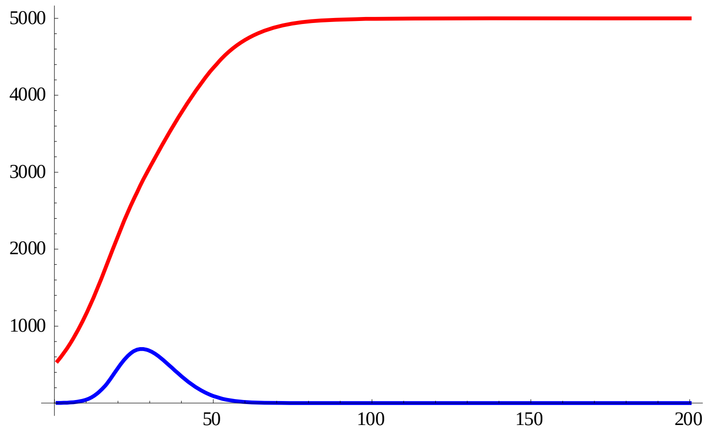
Inspection of Figure 1 reveals that $\gamma_B < \alpha\overline{A}$; thus, both eigenvalues are negative and the fixed point is asymptotically stable. □

The intuition behind Firm A's dominance ('winner takes all') comes from the bracketed terms in (18). For *B* to rise, *B*'s per unit growth rate (γ_B-$\gamma_B B/\overline{B}$-αA) must be strictly positive. When *A* and *B* are small, $\gamma_B(1-B/\overline{B})$ can exceed αA. But once *A* (and therefore αA) are sufficiently large, *B*'s per unit growth rate becomes negative while *A*'s remains positive – hence the first mover advantage. Moreover, the greater is α relative to γ_B (i.e., the smaller is the intercept γ_B/α in Figure 1), the sooner *B*'s growth rate will become negative.

Remark. The increase in *B* is temporary. A rising *B* in the initial stages of competition may have little bearing on the final result over time - the rising user community becomes a misleading predictor of Firm B's ultimate value.

The following simulation in Figure 2 illustrates the point. Initially both *A* and *B* grow, but ultimately Firm A takes all the users and Firm B's user community falls to zero.

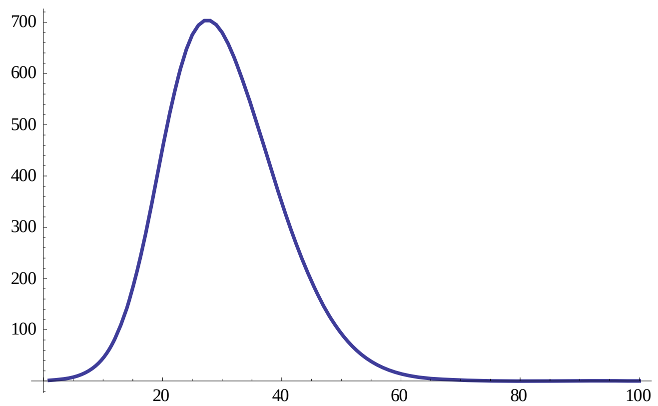
Figure 2. Plot of Both *A* and *B* against Time
(*A* is denoted by the red line, *B* by the blue line)



The user base of Firm A exhibits the familiar S-curve growth (with plateau of 5000 users approximately at time of 90) posited by Cauwels and Sornette (2011), but that of Firm B does not.

Figure 3 highlights the potentially misleading nature of trying to estimate a plateau or carrying capacity – for Firm B the equilibrium size of the user base is zero. Yet statistical estimations of Firm B's carrying capacity or plateau, *based on a small time sample* (from time 0 to 20, for example), are likely to yield a strictly positive plateau.

Figure 3. *B* Plotted against Time



Under what circumstances may Firm *B*'s carrying capacity be reliably estimated?

Consider Case 2.

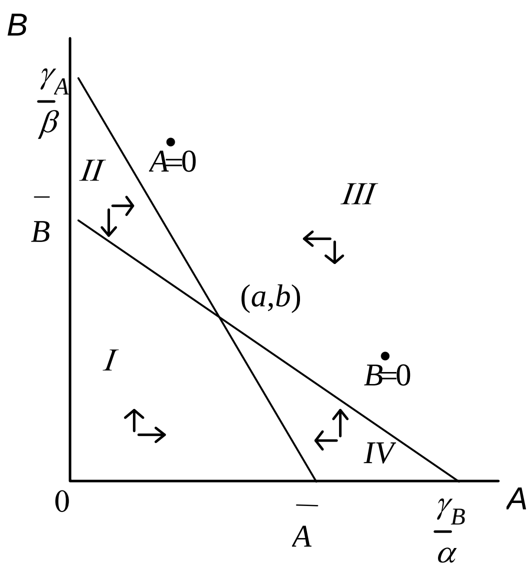
Case 2: $\overline{A} = \overline{B} < \gamma_B / \alpha = \gamma_A / \beta$

PROPOSITION 3 *If* $\overline{A} = \overline{B} < \gamma_B / \alpha = \gamma_A / \beta$, *then* $\lim_{t\to\infty}(A(t), B(t)) = (a,b)$. *The fixed point (a,b) is globally stable.*

Proof: Omitted.

Here the upper limits of the number of potential users is small relative those of Case 1. If $\overline{A}$ and $\overline{B}$ are relatively small, then as *A* and *B* increase, their respective logistic growth rates decline quickly allowing the rival to catch up ground. The potential user communities, $\overline{A}$ and $\overline{B}$, are thus very important in assessing first mover advantages. The first mover advantage accorded Firm A in Case 1 now no longer allows Firm A to dominate the market completely. The phase diagram corresponding to Case 2 is given below. Over time an equilibrium is established at point (*a*,*b*), defining the user plateau that may be estimated for either firm.

Figure 4. Equilibrium User Base less than the 'Carrying Capacity'



CASE 2

Why does Firm *B*'s ultimate user base not fall to zero, despite Firm *A* still being the first mover? The intuition is that, despite Firm A being the first mover, $\overline{A}$ is small, such that *A* tends to $\overline{A}$ quickly. Thus, the growth rate of Firm A's user community falls quickly. Firm B starts to catch up; its competitive impact on Firm A causes the growth rate of Firm A's user community to fall even more. The growth rate eventually becomes negative: *A* falls, further increasing *B* until the equal shares equilibrium is reached.

6. Conclusions

A number of commentators, particularly business consultants, have emphasized the role of virtual communities in generating and transmitting information to potential customers. This has come at a time when economists have been devoting attention to the issue of increasing returns at the firm level (see, for example, Arthur's seminal paper 1994). The theoretical insights from the increasing returns literature, plus the interaction between consumers facilitated by networked technologies, have led to a synthesis in which virtual communities become uniquely valuable to an online firm. Strategy in the electronic business model becomes one of leveraging virtual communities to deepen the relationship between customer and seller, promote customer loyalty and pave the way for a revenue payoff. By developing a user community, firms can also aggregate information about users' transactions on the network, allowing a more effective use of the firm's resources to extract more revenue. The work by Cauwels and Sornette (2011) offers important insights into the usefulness of S-curves and their plateaus or carrying capacities. We have offered an extension to their analysis that suggests that caution by used when attempting to estimate the plateaus.

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