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Testing Hicksian Separability Over Space

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Abstract

If relative prices of goods within a commodity group are constant, Hicksian separability lets the price of a single good represent the group price level. This is relied on by price questionnaires used in household surveys. Methods of estimating demand systems from household survey data also rely on Hicksian separability. Yet this restriction, and its weaker stochastic form under the Generalized Composite Commodity Theorem, remains untested in cross-sections. We use unique data from Vietnam with multiple specifications from within the same food groups to test if within-group relative prices are constant over space. The data firmly reject these restrictions.

Keywords

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demand
prices
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D12; O15

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I. Introduction

Hicksian separability is widely relied upon but rarely examined in the cross-sectional context. This separability requires relative prices of elementary goods within a commodity group to be constant, allowing the price of a single representative good to proxy for the group price level. This type of separability underlies the Composite Commodity Theorem (CCT) and is naturally of interest in studies of product aggregation, where tests with time series data generally reject the restrictions implied by the CCT (for example, Asche, Bremnes and Wessells 1999).

Despite the rejection of Hicksian separability in time series, it is widely relied upon in micro work. Household consumption surveys using a price questionnaire, such as in the World Bank's Living Standards Measurement Study (LSMS), typically choose a single representative specification (for example, a 330ml can of *Coca Cola*) from each group to be priced in local markets. For example, price questionnaires in eight recent household surveys have 87% of food groups with prices available relying on just a single representative specification, while just 13% of groups have multiple goods priced (Table 1).[1] The prices gathered by these surveys may be used to construct spatial and temporal deflators, to form poverty lines, and to estimate demand systems; hence, a wide range of analyses rely on these prices. Yet if Hicksian separability does not hold, multiple goods within each group should be priced, since the structure of within-group relative prices is not constant between locations.

Table 1: Reliance on Hicksian Separability for Price Specifications in Recent Household Surveys

	Food consumption groups with this many items priced				Total number
	≥ Three	Two	One	Zero	of food groups
Vietnam, 1992/93	0	5	24	16	45
Azerbaijan, 1995	0	3	11	7	21
Papua New Guinea, 1996	0	4	17	15	36
Indonesia, 2000	0	1	10	25	36
Tajikistan, 2007	0	2	61[a]	3	66
Panama, 2008	1	6	33	39	79
Nigeria, 2010/11	8	7	72	4	91
Malawi, 2010/11	0	0	21	103	124
Unweighted average (%)	1%	7%	51%	40%	100%

Source: Author's calculations from selected LSMS and IFLS household and community price questionnaires.
[a] Several foods in the price survey were composites such as "Other grain products (e.g. maize, oats, barley)" rather than specific items so it is not clear that a single price specification represented a commodity group.

[1] In these surveys, 41% of food groups have no match to the items in the market price survey.

Even for household surveys without price questionnaires, Hicksian separability is relied on when economists use "unit values" (group expenditures divided by group quantity) as a proxy for market prices to estimate demand systems on cross-sectional data. One must assume that prices of each good in a group move in fixed proportions across locations if unit values are to proxy for group price levels. For example, pork loin is an expensive cut while shoulder is not. If the ratio of loin to shoulder prices is lower in one town than elsewhere, consumers there buy relatively more loin, giving a higher unit value than under fixed price ratios (since the unit value is weighted more towards loin). A need to assume fixed price ratios is explicitly noted by Deaton (1988), who developed the main method for estimating unit value-based demand equations. If the price vector for all the elementary goods within group *G* is decomposed into (i) a scalar term that raises or lowers the price level of all goods in the group across locations and (ii) a reference price vector of the relative price of each good within the group, the inter-area scalar variation must dominate the intra-group variation in relative prices (Deaton 1988). Otherwise, the unit value will not accurately represent the group price level and estimated demand parameters may be biased. But apart from a discussion by McKelvey (2011) of a single food group in a small locality, this key assumption of Deaton's method remains unexamined.

In this paper we use unique data with dual specifications from within the same food groups to test whether within-group relative prices are constant over space. Our dataset combines a standard household survey, the 2010 Vietnam Household Living Standards Survey (VHLSS), with market prices gathered from a spatial cost of living survey fielded in conjunction with the VHLSS. For six food groups (rice, pork, fish, chicken, beef, and fats) prices of two specifications (e.g. both pork rump and pork belly) were observed in up to 1600 different markets. Moreover, price surveyors were equipped with detailed pictures of each specification to ensure that the prices they obtained were for the same item in all locations. We thus have unusually good data with which to test the restrictions implied by the assumption of Hicksian separability over space.

We also test the restrictions implied by the weaker assumption of stochastic Hicksian separability under Lewbel's (1996) Generalized Composite Commodity Theorem (GCCT). If deviations of elementary good prices from their group price index are independent of income and of all the price indices in the demand system, aggregation is still possible for certain demand specifications, such as the Almost Ideal Demand System of Deaton and Muellbauer (1980). In the cross-sectional context, the GCCT requires relative prices to be statistically independent of group price indices. The results from Vietnam reject the restrictions implied by both Hicksian separability and by the weaker stochastic Hicksian separability. Our findings contrast with those in the literature using time series data; usually the restrictions implied by the CCT are rejected but those implied by the GCCT are not and thus provide a basis for aggregation that does not rely on either separable preferences or constant relative prices.[2]

The remainder of the paper is structured as follows. Section II presents a simple model of aggregation and reviews the two types of separability tested here, using the data described in Section III. The testing specification and results are in Section IV, followed by the conclusions.

II. Aggregation and Price Separability over Space

Demand analysts typically face many elementary goods, and hope to consistently aggregate these into a smaller number to enable feasible estimation. But in research with household survey data, the aggregation has already been carried out by the respondent in their report on spending over the recall period, as was noted many years ago by Prais and Houthakker (1955, p.110):

> 'An item of expenditure in a family-budget schedule is to be regarded as the sum of a number of varieties of the commodity each of different quality and sold at a different price.'

Consequently, researchers must work with commodity groups *G* rather than elementary goods *g*. Let p_G be an (observable) aggregate market price index of (unobservable) elementary prices p_g, g=1,…,n and ρ_g are aggregation errors, such that the log elementary prices equal the log aggregate price index $\ln p_G$ plus the error, ρ_g:

$$\ln p_g = \ln p_G + \rho_g \tag{1}$$

If these errors are constant across market locations, Hicksian separability holds, while if ρ_g varies over space but is independent of $\ln p_G$ then stochastic Hicksian separability holds.

In a typical household survey with price questionnaire, just one elementary good *g* is priced per group *G*, making it impossible to observe the error in equation (1). The survey we use has two elementary goods priced per group, allowing the geometric average of these to be used as the group price index, $\ln p_G$. But with just two elementary goods priced out of the many goods in a group, this index still may not be very reliable. We therefore consider another index, the i^{th}

[2] A review of the four sufficient conditions that allow consistent commodity-wise aggregation, and of the empirical evidence for each of these conditions, is provided by Shumway and Davis (2001).

household's unit value (expenditure over quantity) from the consumption recall, $\ln v_{Gi}$ which differs from the group price index due to quality effects and reporting errors:

$$\ln v_{Gi} = \ln p_G + m_{Gi} + v^*_{Gi} \tag{2}$$

where m_{Gi} is a quality effect that is assumed to be negatively correlated with price (consumers react to higher prices by choosing lower quality) and v^*_{Gi} is a pure random reporting error (from errors in reported expenditures or quantities).

If the unit value is used as the group price index, aggregation errors will be correlated with the price index, due to the quality effect. Recalling that the aggregation errors are defined as $\rho_g = \ln p_g - \ln p_G$, if a local average of the unit value (so dropping subscripts) replaces $\ln p_G$,

$$\rho^v_g = \ln p_g - \ln p_G - m_G - v^*_G \tag{3}$$

where ρ^v_g is the aggregation error when using the unit value as the group price index. This error should correlate with the price index because it depends on the quality effect, m_G. Averaging unit values for a local area will not remove this effect, since, all else the same, in a locality with higher prices a lower average quality will be bought. In the method developed by Deaton (1988), these area-wide quality effects are purged from the unit values prior to demand estimation, but Deaton's method depends on the CCT holding. If the data are inconsistent with the CCT, the method of purging quality effects is invalid and the un-purged unit values, in turn, violate the requirements for both the GCCT and the CCT due to the correlation the quality effect induces between aggregation errors and the group price index. Therefore, we emphasize testing of the CCT in the results below, and pay less attention to the testing for stochastic Hicksian separability, since this is always violated by quality effects in unit values.

III. Data Description

In 2010 the lead author designed a price survey for 64 elementary goods, which was fielded in 1,588 communes (almost one-fifth of the total) by the Prices Department of the General Statistics Office (GSO) of Vietnam.[3] These communes were part of the VHLSS consumption survey

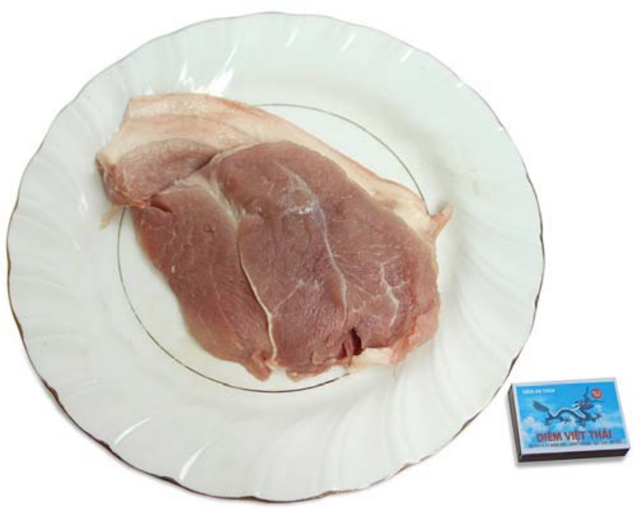
[3] Vietnam's communes are the lowest level administrative unit, averaging about 10,000 people or 2,500 households.

fielded at the same time. To maintain consistency of item specification across areas, enumerators used detailed photographs of each of the 64 goods whose price was required. Figure 1 presents examples of these photographs for two elementary goods in the pork group: rump and belly. The instructions required surveyors to find examples in the market of similar size and quality to what was pictured (with a matchbox used as a scale indicator), and then to weigh them and record prices per metric unit (unless the item was in standard packaging of known weight).

The ability to use these data to examine Hicksian separability resulted from an accident in the design of the consumption recall module for the 2010 VHLSS. This module covers purchases (plus consumption from own-production and gifts) for 53 food and beverage groups. To maintain comparability with earlier years, when diets were less diverse, the food groups had been little changed since the 1990s, but in 2010 the module was revised, switching from a usual month recall to a fixed 30-day recall, and several food groups were to be split to reflect the growing diversity of diets.[4] For example, high quality rice was to be separated from low quality rice, fin fish from shrimp, lard from cooking oil, and so forth. The price questionnaire was implemented with these splits in mind and following the rule of thumb used by similar surveys of pricing a single item per food group (Table 1). But the recall module did not split food groups as planned, and so six major food groups (rice, pork, fish, chicken, beef, and fats – supplying two-thirds of calories) each ended up having two elementary goods priced.

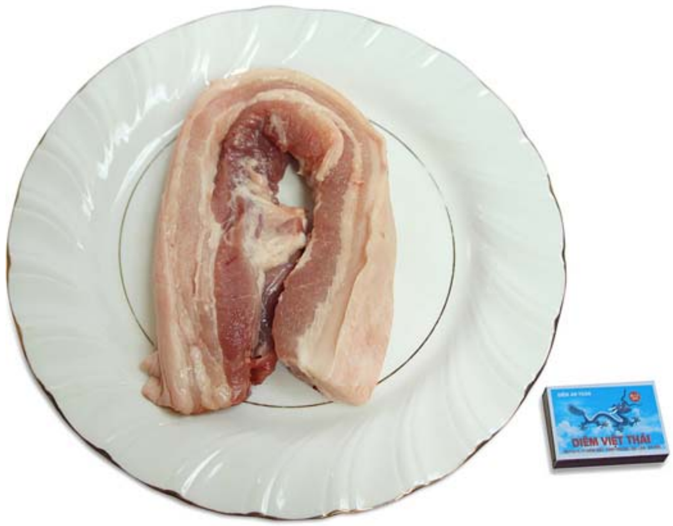
Figure 1: Examples of Photographs Used to Ensure Consistent Price Collection for Items

Panel A: Pork rump, loose, not pre-packaged



[4] Usual month recall is based on reporting the number of months in which the food group is usually consumed by the household, the usual expenditure in those months and the quantity usually consumed.

Panel B: Pork belly, loose, not pre-packaged



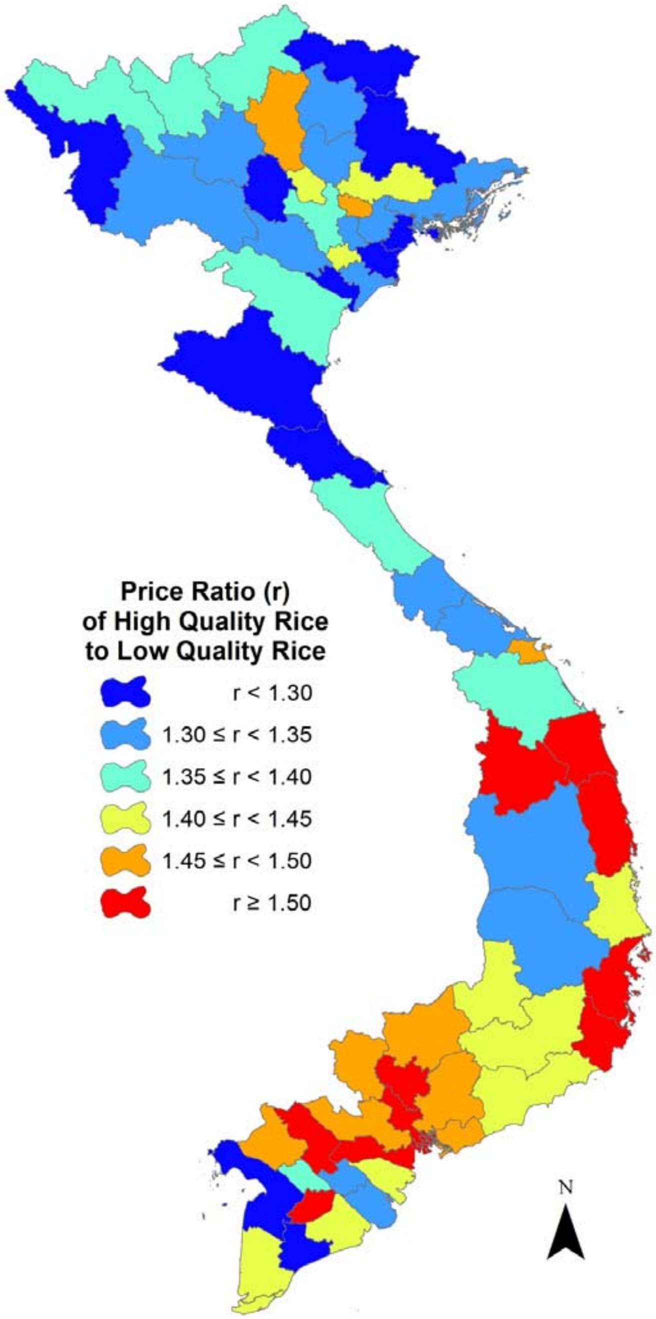
An example of the information provided is shown in Figure 2, which maps the relative price of high quality rice to low quality rice across Vietnam's provinces. The attributes of quality attracting a price premium are color (whiter is dearer), fragrance, stickiness, and taste rather than differences in nutrients, impurities, or proportion of broken grains (<15% for both), and the named rice variety signals this to consumers.[5] On average, high quality rice is 40% dearer than low quality rice but the ratio varies widely over space, with the price survey showing that provincial averages of the price premium range from 19% to 83%. Moreover, there is a distinct geographic pattern, with high quality rice relatively cheaper in the north; the premium averages 33% in the north and 47% in the south. Of the 32 southernmost provinces, 17 have price ratios exceeding 1.45 while 20 of the 31 northernmost provinces have price ratios below 1.35. Since the GSO code numbers provinces from north to south, a simple regression of the price ratio on the province code also shows a statistically significant effect (t=3.2).

There is a good reason for this geographic pattern, which also likely occurs in other settings, and provides a general argument for why Hicksian separability is unlikely to hold over space for many commodity groups. The marketed surplus of rice flows from the south to the north in Vietnam (and from the south to the world market). It costs the same to ship high quality rice and low quality rice, so the addition of a per unit transactions cost should make high quality rice relatively cheaper in the north, which is exactly the pattern shown in Figure 2. This is an

[5] Eamples of the high-quality varieties for the price survey were Bac Huong, Tam Xoan and Jasmine 85, while the low-quality varieties were IR 50404, Khang Dan and Tap Giao.

example of the well-known Alchian-Allen effect of “shipping the good apples out” (Borcherding and Silberberg 1978). Unless transport costs are *ad valorem*, relative prices of high quality and low quality goods that are shipped from a common location (such as the nearest port, for imports, or the nearest large city, for domestic products) are unlikely to be constant over space.

Figure 2: Relative Price Variation Over Space: High Quality and Low Quality Rice



IV. Test Specifications and Results

We begin with a simple test of Hicksian separability that formalizes what is shown in Figure 2, and which informs one main use of price questionnaires in household surveys – the calculation of spatial deflators. If Hicksian separability holds, estimated differences in prices between areas for a given commodity group should be the same, regardless of which elementary good within the group is used to represent the local price level. We therefore estimate:

$$\ln p_{gj} = \sum_k \alpha_k + \beta_1 D + \sum_k \beta_2 D \times \alpha_k + u_{gj} \tag{4}$$

where p_{gj} is price of the elementary good (g=1,2) in the j^{th} commune, D=1 if the specification is for the first elementary good and zero otherwise, α_k represents the fixed effect for region k (showing the percentage difference in price compared with the base region), and u_{gj} is a pure random error.

The key parameter is β_2 which shows if the pattern of regional price differences is sensitive to which of the two elementary goods is used. If we reject $\beta_2 = 0$ for all k, it suggests that using a single representative good to estimate inter-regional price differences may cause bias since a different pattern of areal fixed effects results from using one elementary good rather than the other. Indeed, the results in Table 2 show that for all six food groups, inter-area price differences are sensitive to which elementary good is used, regardless of whether we use provinces (n=63) or districts (n=639) as the geographical level for estimating the α_k fixed effects. In other words, relative prices within food groups must vary over space in Vietnam, to cause this sensitivity of the areal fixed effects to the choice of the elementary good whose price is used.

When we move from spatial deflators to demand estimation, the analysis needs to be at the household-level, using the commune-level market prices without any geographic aggregation. We use a Linear-Approximate AIDS (LA-AIDS) model, which is a demand specification that is compatible with stochastic Hicksian separability. Noting that the budget shares available from a household survey are already group-level aggregates over elementary goods, the model is:

$$w_{Gi} = \alpha_G + \beta_G \ln x_i + \theta_G \ln p_G + \gamma_G \cdot z_i + u_{Gi} \tag{5}$$

where w_{Gi} is the budget share for group G, x_i the total expenditure, and $\mathbf{z}_i$ the vector of characteristics and conditioning variables for the i^{th} household,[6] and p_G is the group G price index (observed at commune level). As shown in Lewbel (1996), the error, u_{Gi} should include any errors ρ_g from aggregating elementary goods into a group price index.

Table 2: Sensitivity of Areal Group-Level Price Fixed Effects to Choice of Elementary Good

Food	Specification of Elementary Good:		Sample	F-test (?2=0 for all k areas)	
Group	First	Second	size	k=63 provinces	k=639 districts
Rice	Low quality (e.g IR50404)	High quality (e.g Bac Huong)	2594	18.5	20.1
Pork	Rump	Belly	3146	20.6	31.1
Beef	Brisket	Rib	2914	60.8	187.1
Chicken	Fresh, battery-raised	Live, free-range	2304	44.8	45.9
Fish	Carp	Fresh-water shrimp	2058	80.0	459.4
Fats	Lard	Neptune cooking oil (500ml)	2784	94.2	763.1

Note: All F-test values are statistically significant at p <0.01 level, and are based on robust variance-covariance matrices. Specifications for all elementary goods are per kilogram prices, unless weight or volume is noted.

We implement three tests with equation (5), with the results reported in Table 3. Since the regression errors include the ρ_g from aggregating prices of elementary goods into a group price index, we generate residuals from a model where the geometric mean of the elementary goods prices is used to measure p_G. The first two columns in Table 3 show the results when these residuals are regressed on the prices of each elementary good. For five of the food groups, there are statistically significant relationships between the residuals and the prices of the constituent elementary goods, with only the beef group residuals showing no relationships. These results suggest that aggregation errors are neither constant, nor independent of the price index.

Since the price index is comprised of only two elementary goods, the residual-based test may not provide very compelling evidence. We therefore implement another test, following the lead of McKelvey (2011), who had two elementary goods (rice varieties) from one food group and simply used the price of one of them (IR64) as the proxy for the group price index, p_G, ignoring the price of the other variety (Cisadane). If the restrictions implied by the CCT hold exactly, it is impossible to estimate equation (5) with the prices of both elementary goods used at once, since they would be perfectly collinear. Even if the restrictions hold only approximately,

6 Specifically, we use (log) household size, the share of the household who are young children, youths, elderly, and migrants, the age, education and gender of the household head, dummy variables for if the household head earns wages, farms, or is self-employed, and non-food budget shares (since this is a conditional demand system).

once the group-level price variation over space is accounted for by the price of one elementary good, there should be no explanatory power coming from the price of the other elementary good. In fact, the results in the third and fourth column of Table 3 show that except for the beef and fish groups, there is significant explanatory power from the prices of *both* elementary goods, in violation of what would be expected under the CCT.

Table 3: LA-AIDS Demand Model Tests of Restrictions Implied by CCT and GCCT

	Relationship between residuals and elementary good prices		Using elementary good prices as the group price index		F-test for equality
Group	First good	Second good	First good	Second good	($\theta_{g1} = \theta_{g2}$)
Rice	0.028	-0.023	0.032	-0.024	17.82***
(n=3888)	(3.57)***	(3.81)***	(3.46)***	(3.63)***	
Pork	0.032	-0.024	0.053	-0.012	42.45***
(n=4722)	(5.86)***	(5.76)***	(9.05)***	(2.45)**	
Beef	-0.003	0.002	-0.001	0.004	2.00
(n=4374)	(1.23)	(1.01)	(0.31)	(2.53)**	
Chicken	0.009	-0.012	0.007	-0.017	18.14***
(n=2565)	(2.68)***	(3.58)***	(1.93)*	(4.81)***	
Fish	0.014	-0.008	-0.002	-0.025	68.91***
(n=3090)	(6.73)***	(5.09)***	(0.82)	(16.43)***	
Fats	-0.000	0.003	0.002	0.009	6.95***
(n=4179)	(0.91)	(1.84)*	(6.47)***	(3.84)***	

Note: Robust t-statistics in ( ), with ***, **, * representing levels of statistical significance of 1%, 5% and 10%.
The first and second elementary good for each group are defined in Table 2.

For the final test, we consider whether the θ_{g1} and θ_{g2} coefficients on the prices of each elementary good in equation (5) are equal. Since most household surveys use the price of just a single elementary good to proxy for the group level price, on the rare occasions when prices of multiple elementary goods are available it is likely that demand analysts chose to work with just one of them as a proxy for the group price index, as did McKelvey (2011). But such a procedure raises the question of whether the estimated demand elasticities are sensitive to which elementary good is used, which motivates a test of $\theta_{g1} = \theta_{g2}$. The results in the last column of Table 3 show that for all food groups except beef, we would reject the equality of the coefficients on the prices of the two elementary goods. In other words, the estimated own-price elasticity of demand for a food group is likely to be sensitive to the choice of which elementary good from within that group has its price used as a proxy for the group-level price index, in violation of what should occur under Hicksian separability.[7]

Does this sensitivity of the estimated elasticities matter, in any practical sense? One use of such elasticities is for calculating the revenue effects of marginal tax changes, as part of the overall social cost of raising one unit of fiscal revenue by increasing the tax (or reducing the subsidy) on food group *G* (see Deaton 1997, pp.326-7 for details). To provide an example of practical effects from the failure of Hicksian separability, we carried out these calculations, alternately using elasticities based on the price of the first, and then the second, elementary good within each group.[8] While rice always emerges as the least attractive candidate for a tax increase (having the highest cost-benefit ratio), there is considerable sensitivity in the ranking of other food groups depending on whether the price of the first or second elementary good is used. For example, pork appears as the second least attractive candidate for a tax increase when we use the price of pork belly to indicate the price level of pork in each commune, but it is the third best (or fourth worst) candidate when the price of pork rump is used as the indicator good.

Finally, we consider the results when unit values (averaged to commune level) are used as the group price index. If the aggregation errors, $\rho_g^v = \ln p_g - \ln v_G$ are added to a unit value variant of equation (5), with v_G replacing p_G, the added errors are statistically significant (p<0.01) for all food groups. We also directly test the independence of the unit values and the aggregation errors, rejecting the null of independence at the p<0.01 level in all cases.[9]

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7 The own-price elasticity from a budget share equation like (5) depends not only on θ_G, but also on group budget shares and the response of quality (the unit values) to price (McKelvey 2011). Unless different θ coefficients from one elementary good versus another are exactly offset by different responses of quality to each of those elementary good prices, the own-price elasticities will differ since the same budget share is used with both goods.

8 Full details on these elasticity and welfare cost-benefit ratio calculations are available from the authors.

9 Full details on these results using unit values are available from the authors.

V. Conclusions

Hicksian separability is widely relied upon in the cross-sectional context. Yet, our tests show that at least for Vietnam, within-group relative prices vary significantly over space. Moreover, the Alchian-Allen effect suggests this should be the expected pattern, despite survey designers and demand models from unit values relying on an assumed absence of such effects. Few household surveys price multiple goods within each group so this simple restriction has not previously been able to be tested. The current results suggest the need for repeating our tests elsewhere.

Our results also suggest greater effort is needed to gather price data that are spatially and commodity-wise disaggregated. Currently, statistical agencies prioritize nominal living standards data over price data, even in poor countries where costly internal transport and lack of dominant brands and retail chains make it is implausible that prices for a given good are everywhere the same (Gibson, 2013). Instead of the widely used strategy of pricing a single elementary good per commodity group, household surveys should gather prices on multiple goods within each group, since the structure of within-group relative prices apparently is not constant between locations.

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