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The Impact of Citation Timing: A Framework and Examples

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Abstract

The literature on research evaluation has noted important differences in citation time patterns between disciplines, high and low ranked journals and types of publications. Delays in the receipt of citations suggest that the diffusion of knowledge following discovery is slower, and may thus be associated with a decrease in the impact of research. This paper provides a framework for the comparison of different citation time patterns. Using principles drawn from the literature on stochastic dominance we show that comparisons of time patterns can be based on the general characteristics of cost of delay functions. When a particular function is used to represent the cost of delay, the magnitude of the impact of differences in citation time patterns can be assessed using simple exponential discounting. We demonstrate the application of this framework in assessing different citation time patterns by applying it to comparisons of 10-year citation records for: leading journals in economics, different business subject areas, journals in economics compared with those in neuroscience and the research output of individual economists.

Keywords

citations
citation time patterns
discounting citations
research measurement

JEL Codes

A19, C81, J24

1. Introduction

Citation time patterns have been the subject of considerable research. It has been noted that some disciplines attract more citations and earlier citations than others (for example, Evidence 2007), that there is considerable variation between the citation patterns associated with individual papers (for example, Levitt and Thelwall 2008), that the time pattern of citations is different between high and low ranked journals in the same discipline (Anderson and Tressler 2016) and that citations histories can be used to classify papers by type (for example, Redner 2005).

The importance of these differences has been heightened by the direct and indirect use of bibliometrics in National Research Assessment Exercises (OECD 2010). This has raised the issue of the reliability of citation data when it is collected over short periods (Tressler and Anderson 2012). The timing of citation patterns is central to question of whether short-term citation patterns can be used to predict long-term citations and thus research impact (see, for example, Burns and Stern 2015 and Stern 2014). Mechanistic models have also been used to predict citation outcomes and to represent the citation dynamics of individual papers based on key parameters (see, for example, Wong, Song and Barabási 2013).

In a related literature stemming from the seminal work in economics of Jaffe, Traitenberg and Henderson (1993), citations to patents have played a significant role in the empirical study of the diffusion of knowledge following discovery. This literature has considered the implications of delays in citations and empirical models of the pattern of patent citations over time.[1]

Articles, journals and academic areas differ in the levels and patterns of citations that are generated over time. Here we seek to provide a framework that enables these patterns to be compared. In the context of the analysis considered in this paper the level and pattern of citations provides an indicator of the contribution to knowledge represented by a particular paper, all papers in a journal, all journal articles published in a particular academic area, or all articles published by an individual.[2]

It is clear from the literature cited above that the timing of the flow of citations does matter. The knowledge diffusion approach is based on the premise that the earlier a work is cited the greater its potential impact on other knowledge creation. In considering flows of consumption or income over time, the existence of financial markets provides a clear rationale for using simple present value calculations to capture the influence of time.[3] In

[1] See for example Jaffe *et al.* (1993) and Mehta, Rysman and Simcoe (2010). While citations to patents have some of the characteristics of academic journal citations, in other respects they are very different. For example Jaffe *et al.* note that citations to preceding patents have value implications for the citing patent, thus removing the likelihood of frivolous citations.

[2] These indicators can of course be biased (Kim, Min and Zimmermann (2011) or subject to manipulation and error (Archambault and Lariviere 2009).

[3] Exponential discounting can also ensure time consistency in intertemporal decision making, see, for example, Gollier (2001).

dealing with citations as indicators of knowledge development we seek a more general framework for the consideration of the influence of time.[4] In the literature on the economics of uncertainty, stochastic dominance has been useful in providing a theoretical foundation for comparisons of probability distributions. It also provides the basis for the definition of risk in economics. Stochastic dominance concepts have been applied in many contexts in economics, for example: in the characterisation of inequality and poverty in relation to distributions of income (Atkinson 1970, 1987 and Davis and Hoy 1995), economic organisation (Sah and Stiglitz 1986) and the economics of the evaluation of citations received by a particular individual (Ravallion and Wagstaff 2011). Jackson and Rogers (2007) use stochastic dominance concepts in describing efficiency results for social networks including networks of citations emanating from single paper. Here stochastic dominance concepts are used to provide a foundation for the characterisation of some differences in the distribution of citation patterns over time.

Citations are only one indicator of the contribution to knowledge derived from particular publications or sets of publication. Clearly there are many factors that also provide indicators of the likely contribution to knowledge from a particular publication such as: the research record of the authors, the journal in which the research is published, the length of the paper, etc. Similarly, there are a number of factors that will influence the likely contribution to knowledge that is indicated by a particular citation such as: the research record of the citing author, the quality of the journal in which the citation is made, where in the history of citations the particular citation occurs, etc. Here we consider only the influence of time on the contribution to knowledge that is indicated by citations. We concentrate on this factor because we are interested in differences in the time patterns of citations for different journals, papers in a subject or individuals.

1. A Framework for the Comparison of Citation Timing Patterns

To provide a simple framework we consider research published at time t_0 and the pattern of citations to that research received by time t_n.[5] Let the knowledge gained as indicated by these citations be represented by

$$K(t_0, t_n) = k_{t_0} h_{t_0} + \cdots + k_{t_n} h_{t_n},$$

where k_{t_i} is the contribution to knowledge of a single cite at time t_i, and, h_{t_i} the number of cites at time t_i.[6]

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[4] Although this paper centres on interpreting citations as an indicator of knowledge creation, citations to papers can also modelled as being the result of random and deliberate social interaction in networks, see e.g. Jackson and Rogers (2007), or as the outcome of strategic decision making, see, for example, Kim, Min and Zimmermann (2011).

[5] In the discussion, citations refer to: citations to articles; all articles in a journal; or to articles published by an individual in t_0. Of course this could be citations per page or citations per page divided by the number of authors.

[6] Clearly even this general formulation is restrictive as we are implicitly assuming that the contribution to knowledge indicated by a citation at time t_i is independent of the patterns of citations in the period prior to or after that time period.

It is important to note that this formulation implies that the knowledge contribution from a particular cite does not depend on the number of other cites occurring at the same time and that all cites result in the same indication of contribution other than through the influence of time.[7] This is a simplification, but while the probability of a cite may well depend on the number of cites being made by others, it is not so clear why this should impact on the contribution to knowledge that a particular cite signals. With few cites occurring at the same time discovery might be signalled, with many cites, momentum in the discovery of ideas might be signalled.

The treatment of the signal provided by a cite as independent of the cites occurring around it is consistent with the way in which cites are treated in assessing the contributions of journals through impact factors or impact factor related metrics. If the value indicated by a particular cite depended on the number of cites made to a particular article, then to properly assess the value of a journal using cites it would be necessary to consider the distribution of cites across articles in an issue of a journal. The same would be true in assessing the value of the contribution made by a particular individual who has published a number of works.
Rewriting the above treating time as continuous and letting $t_n=t_1$

$$K(t_0, t_1) = \int_{t_0}^{t_1} k(t)h(t)dt,$$

where $k(t)$ is the contribution to knowledge of a single cite at time t and $h(t)$ the rate at which citations are occurring at time t. For ease of exposition we will treat time as continuous in what follows, although we will note key results for the discrete time case in which the periods are of equal length. Let N be the total number of cites received over the period t_0 and t_1, then

$$K(t_0, t_1) = \int_{t_0}^{t_1} k(t)N\frac{h(t)}{N}dt = N\int_{t_0}^{t_1} k(t)f(t)dt,$$

where $f(t)$is the proportional rate at which cites occur at time t. Alternatively, $f(t)$ could be viewed as the probability density for cites occurring between t_0 and t_1 where the timing of citations is thought of as a random variable. The equation above indicates that the contribution to knowledge $K(t_0,t_1)$ can be viewed as the product of the number of cites and a term that represents the time adjusted contribution to knowledge $\int_{t_0}^{t_1} k(t)f(t)dt$.

To provide an alternative representation, let the contribution to knowledge of a single cite at time t be $k(t)=D-d(t)$, where D can be thought of as the contribution to knowledge a cite would have if it had occurred at time t_0 and $d(t)$the decline (or possibility increase) in the contribution to knowledge if the cite occurs at time t. Then

$$K(t_0, t_1) = N\int_{t_0}^{t_1} \big(D - d(t)\big)f(t)dt = N(D - \int_{t_0}^{t_1} d(t)f(t)dt).$$[8]

[7] The analysis could be applied to weighted cites allowing account to be taken of the quality of the citing journal as an indicator of the likely contribution to knowledge.

In the equation above $\int_{t_0}^{t_1} d(t)f(t)dt$ represents the expected decline in knowledge resulting from the distribution of citations over time. If $d'(t) > 0$ for all t then this would suggest that an increase in t always results in a decline in the contribution to knowledge, since the impact is delayed in time and the influence gained in knowledge has less time to develop. This would seem to be a reasonable assumption to impose.

Although less obvious, another plausible assumption is that $d''(t) < 0$ for all t. Here $k(t)=D-d(t)$, thus assumptions that $d'(t) > 0$ and $d''(t) < 0$ correspond to the assumptions that $k'(t) < 0$ and $k''(t) > 0$, or that as time t increases the contribution to knowledge indicated by a citation decreases, but at a decreasing (numerically increasing) rate, or the cost of delay is increasing but at a decreasing rate. With normal exponential discounting $k(t) = De^{-r(t-t^0)}$, where D is the value of a cite at time t^0. Thus, the assumption that $k''(t) > 0$, or $d''(t) < 0$, is consistent with normal discounting practice. That is the decline in value in early time periods is greater than the decline further in the future, thus a one year delay from year zero is treated as having a much larger impact on value than a one year delay from year 10. In terms of cites this corresponds to the assumptions that a delay in a cite in early years after publication implies a greater impact on the likely development of knowledge than a delay in later years.

In the context of expected utility analysis Rothschild and Stiglitz (1970) show the concavity of the utility function $u(x)$, where x is the random variable of interest, corresponds to a dislike for mean preserving spreads in the probability distribution of x. This then provides a natural definition of risk based both on the spreading of probability weight around a constant mean, and the preferences of risk averse decision makers. Similarly, in this context, the assumption that $d''(t) < 0$ for all t implies that a change in the time distribution of citations such that a higher proportion of the cites occur in early and later years while the mean time remains the same would decrease the expected value of the decline in knowledge, that is, increase the gain in knowledge as indicated by the cites received. An increase in early and late cites would indicate that the contribution was having a continued impact rather than a concentrated one.[9]

Applied to the framework above, the theory of stochastic dominance enables distributions of the proportional rate of cites to be compared.[10] Consider two distributions $f(t)$ and $g(t)$ and let $F(t) = \int_{t_0}^{t} f(t)dt$ and $G(t) = \int_{t_0}^{t} g(t)dt$ for $t \epsilon [t_0, t_1]$, then:

1. If $F(t) \leq G(t)$ for all $t \epsilon [t_0, t_1]$ and strictly less from some t, then $F(t)$ is said to dominate $G(t)$ in the first degree, i.e. in this sense cites will always occur later with $F(t)$

8 Note that $\int_{t_0}^{t_1} f(t)dt = 1$.

9 Redner (2005) defines 'unpopular' papers as papers '…that are cited soon after publication, if at all, and then disappear.'

10 The classic article on first and second degree stochastic dominance as used here is Hadar and Russell (1969), see also Gollier (2001), Chapter 3.

than $G(t)$ as the proportion of cites occurring up to t is always less for $F(t)$ than $G(t)$. For all functions $d(t)$ with $d'(t) > 0$ the expected delay in the contribution to knowledge $\int_{t_0}^{t_1} d(t)f(t)dt$ is always greater with $f(t)$ than $g(t)$ and thus the distribution of citations always suggests that knowledge is devalued more.

These conditions can be extended to the discrete time period case.[11] For example, assume that citations are observed at the end of each of a finite number of n years, and let f_i and g_i be the proportion of cites received in year i. Let $F^r = \sum_{i=1}^{r} f_i$ and $G^r = \sum_{i=1}^{r} g_i$ be the proportion of cites received by time r, where $r \leq n$, then the patterns of citations represented by the f_i ,i=0 to n dominates that represented by g_i , i=0 to n in the first degree if $F^r \leq G^r$ for all years $r \leq n$ and is strictly less for some years.

2. If $\int_{t_0}^{t} F(t)dt \leq \int_{t_0}^{t} G(t)dt$ for all $t \epsilon [t_0, t_1]$ and strictly greater for some t, then $F(t)$ is said to dominate $G(t)$ in the second degree. It follows that for all functions $d(t)$ with $d'(t) > 0$ and $d''(t) < 0$ $f(t)$ results in a greater expected delay than $g(t)$ and thus the distribution of citations always suggests that knowledge is devalued more. For the discrete case, the patterns of citations represented by the f_i ,0=1 to n dominates that represented by g_i , 0=1 to n in the second degree if $\sum_{i=1}^{r} F^i \leq \sum_{i=1}^{r} G^i$ for all $r \leq n$ and strictly less for some t.

Table 1 describes two hypothetical comparisons over a ten year time horizon. In the first comparison the distribution of cites f comes unambiguously later than that of g which has the same pattern of cites, but starting one year earlier. In this case condition 1 is clearly satisfied, i.e., for all t the cumulative percentage of cites is weakly greater with g than f and strictly greater for some t. In the second comparison the distribution of cites is clearly less spread with h than with g case even though the mean time at which cites occur is the same. Here h dominates g in the second degree, and thus the contribution to knowledge is greater with g than h as delay costs are less with g.

Table 1: Some Examples of Distributions of Cites

Time	1	2	3	4	5	6	7	8	9	10
f_i	0	0	¼	¼	¼	¼	0	0	0	0
g_i	0	¼	¼	¼	¼	0	0	0	0	0
F^r	0	0	¼	½	¾	1	1	1	1	1
G^r	0	¼	½	¾	1	1	1	1	1	1
g_i	0	¼	¼	¼	¼	0	0	0	0	0
h_i	0	0	½	½	0	0	0	0	0	0
G^i	0	¼	½	¾	1	1	1	1	1	1
H^i	0	0	½	1	1	1	1	1	1	1
$\sum G^i$	0	¼	¾	1 ½	2 ½	3 ½	4 ½	5 ½	6 ½	7 ½
$\sum H^i$	0	0	½	1 ½	2 ½	3 ½	4 ½	5 ½	6 ½	7 ½

[11] Here citations are treated as if they are received at the end of a finite number of equally sized time periods. This ignores the pattern of citations received over the single period.

This treatment of time corresponds to the usual present value assessment in the special case in which $d(t) = D - De^{-rt}$ for some D as above representing the value of a cite at time t_0 and discount rate r. For this function $d'(t) = r\bar{D}e^{-rt} > 0$ and $d''(t) = -r^2 D\bar{k}e^{-rt} < 0$ as assumed above. Setting $t_0 = 0$, in this case

$$K(0,t_1) = N\int_0^{t_1}\big(D - d(t)\big)f(t)dt = N\int_0^{t_1}(D - (D - De^{-rt}))f(t)dt$$
$$= N\int_0^{t_1} De^{-rt}f(t)dt.$$

Thus $K(0,t_1)$ represents the number of cites over the total period multiplied by the present value of cites, i.e. the proportional flow of cites discounted at the constant discount rate r. For the discrete case, the patterns of citations represented by the f_i , i=0 to n would have lower present value than that represented by g_i, i=0 to n if $N\sum_{i=1}^{n} De^{-ri} f_i \le N\sum_{i=1}^{n} De^{-ri}\, g_i$.

If this particular value decline function is used it is possible to compare any two distributions of cites in terms of the number of cites occurring over a given time period N, the value of single cite D and the discount factor applying to each distribution, $\sum_{i=1}^{n} e^{-ri} f_i$ and $\sum_{i=1}^{n} e^{-ri}\, g_i$. In this case the ratio of the value of cites would be independent of the number of cites. Without loss of generality we treat the value of an undiscounted cite D=1.

A cite is a signal of knowledge creation through the recognition of the contribution made by a piece of research. Having been recognised, the contribution may also have an impact on future knowledge creation. In a situation in which a simple contribution is made that might be easily discovered or replicated by other researchers, the relevant discount rate might be quite high. In other cases in which a significant theoretical contribution is made it is possible that the future direction of a discipline might be affected. In this case the relevant discount rate might be very low.

One example of the discounting of citations in the economics discipline is the use of discounted impact factors and recursive discounted impact factors by the CitEc project that is part the Research Papers in Economics (RePEc) collaboration.[12] The discount factor that is used here is one divided by the age of the citing paper in years with the current year having a value of one. It is important to note that this is not discounting of the delay in a citation from publication as considered here, but discounting of the age of the cite. Thus a citation from a paper one year ago receives a weight of 0.5 and two years ago 0.33.[13] If this discounting

12 In a broader context Jin, Liang, Rousseau and Egghe (2007) and Holden, Rosenberg and Barker (2016) also suggest discounting by dividing by the age of the article, but not the time between the publication of the cited article and the cite. Järvelin and Persson (2008) also consider discounting based on the age of the cite using a logarithmic function.

13 See https://ideas.repec.org/top/top.series.discount.html or Zimmermann (2013).

method was applied to the delay between publication and citation, then a cite received one year after publication would receive a weight of 0.5 and two years after publication 0.33. While this would not be consistent with exponential discounting as used to represent the cost of delay here, the implied discount rates are very high, with r=0.693 for a cite one year out, r= 0.543 two years out, r=0.462 three years out and r=0.255 ten years out.[14] In terms of the framework used in this paper a RePEc type delay function could be written as $d(t) = D - 1/(t+1)$. For this function $d'(t) > 0$ and $d''(t) < 0$ as assumed above.

In context of the economics of uncertainty a function with the equivalent functional form as the exponential delay function used here is the utility function $u(w) = A - be^{-rw}$, where w is uncertain income or wealth. This is referred to as the constant absolute risk aversion utility function. In this case the cost of risk, or risk premium, is not influenced by an additive shift in the distribution of wealth, that is, by a simple constant increase or decrease in wealth in all states of the world.[15] In the case of cites it would imply that the impact of the spread of cites over the period around the same mean time would not be affected by where that distribution of cites was placed in time.

3. Application of the Framework

In this section we make use of data on 10-year citation patterns collected as part of two other studies to demonstrate the ways in which the methods developed above can be used to determine both general patterns in citation timing and the importance of the differences in timing. Tressler and Anderson (2012) considers citation lags for New Zealand economists and suggests that they make it difficult to rely on citation counts as meaningful measures of research output in time-limited research assessments. Anderson and Tressler (2016) uses citation capture rate data to descriptively compare citation timing patterns between social sciences, business subjects and science and between leading and lower ranked journals in economics. They show that short-term citation counting favours science over social science and within economics, lower quality journals overs higher quality ones.

Example One:
Comparisons of Citation Patters for Journals in Economics

We consider first 10 year citation patterns for research published in a number of leading journals in economics: *Journal of Political Economy* (*JPE*), *Quarterly Journal of Economics* (*QJE*), *American Economic Review* (*AER*), *Review of Economic Studies* (*RES*) and *Econometrica* (*EM*) and for comparison purposes two 'letters' journals *Applied Economics Letters* (*AEL*) and *Economics Letters* (*EL*). Letters journal are often assumed to provide the opportunity for research results to be disseminated rapidly. Rapid dissemination is a stated aim of *Economics Letters*. All citation data is collected from Thomson Reuters/WoK.

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[14] This form of discounting is less severe in comparison to exponential discounting as the paper age becomes higher.

[15] See for example Kreps (1990), pp.84-85.

Papers eligible to receive citations are published in the journals considered in 2003 while citations to these papers are from all journals in the WoK databases from 2003 to 2012.[16] Table 2 presents the basic data, showing the percentage cites to articles published in 2003 over each of the ten years. On average less than one percent of cites are received in the first year rising to almost 16% in Year 10. Cumulative percentage cites and the sum of cumulative percentage cites as required for the stochastic dominance based comparisons are also provided.

Based only on the assumption that delay reduces the value of cites (i.e. for any delay function *d(t)* with $d'(t) > 0$), the cost of the delay in the uptake of knowledge, as indicated by citations, is unambiguously greater for the *QJE* compared with the *JPE*, the *QJE* compared with the *AER*, the *EL* compared with *AER* and the *RES* compared with *AEL*. Alternatively, let $f \succcurlyeq_d^1 g$ indicate that the distribution of citations for f has unambiguously more delay than the distribution g in the first degree, then $QJE \succcurlyeq_d^1 JPE,\ QJE \succcurlyeq_d^1 AER,$ $EL \succcurlyeq_d^1 AER$ and $RES \succcurlyeq_d^1 AEL.$ If, in addition, the assumption that an increase in the spread of citations would increase value or decrease delay, that is, $d'(t) > 0$ and $d''(t) < 0$, then the delay in the uptake of knowledge is unambiguously greater for the *AER* in comparison to the *JPE* and *QJE* relative to the *EM*. Thus, if $f \succcurlyeq_d^2 g$ indicates that the distribution of citations for f has unambiguously more delay than the distribution g in the second degree, then $JPE \succcurlyeq_d^2 AER$ and $QJE \succcurlyeq_d^2 EM.$ It follows that $QJE \succcurlyeq_d^1 JPE \succcurlyeq_d^2 AER$ for all *d(t)* with $d'(t) > 0$ and $d''(t) < 0$.

When present values are used in order to assess the impact of delay, it is possible to compare all journals. The discount factors ($\sum_{i=1}^{n} e^{-ri} f_i$) for the journals for a number of discount rates are shown in Table 3. Differences in these numbers for a given discount rate represent the impact of differences in citation timing between a pair of journals as a proportion of the undiscounted value of a single cite. Here, these differences seem quite small, less than 4% for all discount rates and often less than 1%. Of course the impact of these differences as a percentage of the discounted values is much higher. Thus, for example, at discount rate of 10%, the maximum difference in discount factors is between the *QJE* and *AEL* at 3.2% of the undiscounted value of a cite (equal to 1.0) and 5.8% of the discounted value of a cite in the *AEL*. However, at a discount rate of 40%, the maximum difference in discount factors is 2.9% of the undiscounted value of a cite and 23% of the discounted value.

For all discount rates under 40 percent, the rank of the journals from the lowest discount factor to the highest remains the same: *AEL*, *RES*, *AER*, *JPE*, *EL*, *EM* and *QJE*. *Applied Economics Letter* gathers cites the fastest and the *Quarterly Journal of Economics* the slowest.

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[16] The increase in the number of JCR listed economics journals over the 10 year period would have also increased citations over this period.

Table 2: Citation Patterns for Economics Journals, 10YR ISI Cites to 2003 Publications

Journal		2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	Cites/ Paper (C)	PV C r=0.1
JPE	% Cites *f*	0.66	2.49	5.92	8.60	10.15	11.32	14.65	15.50	14.98	15.74	50.69	28.72
	Cum. % Cites F	0.01	0.03	0.09	0.18	0.28	0.39	0.54	0.69	0.84	1.00		
	Sum Cum. % Cites $\sum F$	0.01	0.04	0.13	0.31	0.58	0.97	1.51	2.21	3.05	4.05		
Q J E	% Cites *g*	0.45	2.25	5.06	7.00	8.24	12.08	14.25	15.58	17.37	17.70	78.58	43.40
	Cum. % Cites *G*	0.00	0.03	0.08	0.15	0.23	0.35	0.49	0.65	0.82	1.00		
	Sum Cum. % Cites $\sum G$	0.00	0.03	0.11	0.26	0.49	0.84	1.33	1.98	2.80	3.80		
AER	% Cites *h*	0.67	3.19	5.98	7.93	9.81	12.55	14.01	15.45	15.03	15.36	40.55	23.07
	Cum. % Cites *H*	0.01	0.04	0.10	0.18	0.28	0.40	0.54	0.70	0.85	1.00		
	Sum Cum. % Cites $\sum H$	0.01	0.05	0.14	0.32	0.60	1.00	1.54	2.24	3.08	4.08		
RES	% Cites *j*	0.41	2.84	5.62	8.66	10.93	14.12	13.81	14.64	15.15	13.81	52.43	29.97
	Cum. % Cites *J*	0.00	0.03	0.09	0.18	0.28	0.43	0.56	0.71	0.86	1.00		
	Sum Cum. % Cites $\sum J$	0.00	0.04	0.13	0.30	0.59	1.01	1.57	2.29	3.15	4.15		
EM	% Cites *k*	0.75	2.64	5.39	6.79	9.07	11.12	14.46	16.25	15.65	17.88	42.05	23.43
	Cum. % Cites *K*	0.01	0.03	0.09	0.16	0.25	0.36	0.50	0.66	0.82	1.00		
	Sum Cum. % Cites $\sum K$	0.01	0.04	0.13	0.29	0.53	0.89	1.39	2.06	2.88	3.88		
EL	% Cites *m*	0.61	3.10	5.83	7.24	7.94	10.95	15.79	14.33	16.78	17.43	8.83	4.94
	Cum. % Cites *M*	0.01	0.04	0.10	0.17	0.25	0.36	0.51	0.66	0.83	1.00		
	Sum Cum. % Cites $\sum M$	0.01	0.04	0.14	0.31	0.55	0.91	1.42	2.08	2.91	3.91		
AEL	% Cites *n*	0.47	3.26	6.52	11.34	11.65	12.73	14.44	12.89	13.51	13.20	3.25	1.90
	Cum. % Cites *N*	0.00	0.04	0.10	0.22	0.33	0.46	0.60	0.73	0.87	1.00		
	Sum Cum. % Cites $\sum N$	0.00	0.04	0.14	0.36	0.69	1.15	1.76	2.49	3.36	4.36		
Ave Percent Cites		0.58	2.83	5.76	8.22	9.68	12.13	14.49	14.95	15.50	15.87	39.48	

Table 3: Discount Factors over 10 Years for Economics Journals

	r=0.01	r=0.05	r=0.1	r=0.2	r=0.3	r=0.4
JPE	0.94	0.75	0.57	0.34	0.22	0.14
QJE	0.94	0.74	0.55	0.32	0.20	0.13
AER	0.94	0.75	0.57	0.34	0.22	0.15
RES	0.94	0.75	0.57	0.34	0.22	0.15
EM	0.94	0.74	0.56	0.33	0.21	0.14
EL	0.94	0.74	0.56	0.33	0.21	0.14
AEL	0.95	0.76	0.58	0.36	0.23	0.16

Table 2 also shows the discounted value of 10 year citations per paper for each journal at a discount rate of 10%. If journals are ranked by the present value of 10 year citations per paper for all discount rates under 17% the ranking coincides with a simple ranking based on citations per paper. For discount rates above 18%, the order of the *AER* and *EM* reverses. Here the 3.4% difference in cites per paper is more than counterbalanced by a 3.6% difference in the discount factor. Since the differences in citations per paper for other journals is large, for discount rates up to 90% the use of the present value of cites does not result in further changes in the ranking of journals.

As shown in Table 3 the discount factor for *EL* is equal to or lower than that for all other journals except the *QJE* at all the discount rates shown. It follows that there is at least as much delay in the receipt of citations for *EL* than all other journals except the *QJE*. Thus this citation data does not suggest that *Economics Letters* results in the rapid dissemination of new ideas. In contrast, the discount factor for *AEL* is greater than the discount factor for all other journals at all discount rates, suggesting that for this data there is evidence of greater dissemination of knowledge through publication with the *Applied Economic Letters* than the other journals considered.

The present value comparisons of journals made above are based on the assumption that the relevant discount rate to apply is the same for all journals. There might be good reason to apply different discount rates to different journals. For example, it could be argued the contributions made in lower ranked journals may be less likely to have a long term impact on the development of knowledge and more likely to be discovered by others in a relatively short time period. Thus there could be a case for using higher discount rates for lower ranked journals.

Example Two:
Comparisons of Citation Patterns for Business Subject Areas

In this section we use the framework developed above to compare the citation patterns for different business subjects. In particular we look at citation patterns for a number of Thomson Reuters/Web of Science (WoS) Journal Citation Report subject categories relevant to a business school: Business (*Bus*), Business Finance (*Bus Fin*), Communication (*Com*), Industrial Relations and Labour (*Ind Rel & Lab*), Information Science and Library Science (*If*

Sc &Lib Sc) and Management (*Mgt*). As above, all citation data is collected from Thomson Reuters/WoK. Papers eligible to receive citations in each discipline category are those published in the journals considered in 2003, whereas citations to these papers are from all journals in the WoK databases from 2003 to 2012. In Table 3 we report citation patterns for all publications in the subject areas considered in 2003 over the following 10 years. As noted in the table, on average these subject areas have less than 1% of citations received in the first year, rising to 15.47% in year nine and 15.23% in year 10.

In contrast to the results for journals in economics, stochastic dominance based comparisons of the citation patters lead to an almost complete unambiguous ranking of these subject areas in terms of delay patterns. Here, for any delay function *d(t)* with $d'(t) > 0$, the cost of the delay in the uptake of knowledge is unambiguously greater for *Mgt* than all other subjects; *If Sc & Lib Sc* is unambiguously less than *Bus*, *Bus Fin*, *Econ* and *Ind Rel & Lab*; *Ind Rel and Lab* is unambiguously less than *Bus*, *Bus Fin* and *Econ*; *Econ* is unambiguously less than *Bus,* but greater than *Com*; *Com* is less than *Bus* and *Bus Fin*; and *Bus Fin* is less than *Bus*.

In addition, for any delay function *d(t)* with $d'(t) > 0$ and $d''(t) < 0$, *Econ* has unambiguously greater delay than *Bus Fin,* and *Com* unambiguous greater delay than *If Sc & Lib Sc*. Only for comparison of *Com* and *Ind Rel and Lab* are rankings not possible. Putting these pairwise rankings together using the notation above it is possible to conclude that either:

$$Mgt \succcurlyeq_d^1 Bus \succcurlyeq_d^1 Econ \succcurlyeq_d^2 Bus\ Fin \succcurlyeq_d^1 Com \succcurlyeq_d^2 If\ Sc\ \&\ Lib\ Sc,$$

or

$$Mgt \succcurlyeq_d^1 Bus \succcurlyeq_d^1 Econ \succcurlyeq_d^2 Bus\ Fin \succcurlyeq_d^1 Ind\ Rel\ \&\ Lab \succcurlyeq_d^1 If\ Sc\ \&\ Lib\ Sc.$$

When present values are used to assess the impact of delays, then for all discount rates less than 24%, cites in *Com* journals involve less delay than in *IR Rel & Lab*, but for discount rates above 25% the order is reversed.

Table 4 also shows the ranking of subjects in terms of citations per paper. If the citations per paper are discounted using a discount factor of 10%, then the subject ranking by discounted citations per paper are the same as for undiscounted citations per paper. The ranking remains the same for discount rates under 36%. At a discount rate of 36%, the 16.2% difference in the discount factor between Business Finance (BF) and Information Science and Library Science (IS) (representing differences in delay) outweighs the 16.1% difference in citations per paper. Thus, comparisons of subjects in terms of citations patterns is a little less sensitive to the timing of citations than was the case for economics journals.

Table 4: Citation Patterns for Business Subjects, 10YR ISI Cites to 2003 Publications Journals

Subject		2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	Cites/ Paper (C)	PV C r=0.1
Bus	% Cites f	0.45	2.23	4.68	6.73	8.87	12.34	15.63	16.57	16.25	16.26	27.50	15.24
	Cum. % Cites F	0.00	0.03	0.07	0.14	0.23	0.35	0.51	0.67	0.84	1.00		
	Sum Cum. % Cites $\sum F$	0.00	0.03	0.10	0.25	0.48	0.83	1.34	2.01	2.85	3.85		
Bus Fin	% Cites g	0.83	3.31	6.07	8.17	8.96	12.29	14.10	15.26	15.53	15.48	17.44	9.92
	Cum. % Cites G	0.01	0.04	0.10	0.18	0.27	0.40	0.54	0.69	0.85	1.00		
	Sum Cum. % Cites $\sum G$	0.01	0.05	0.15	0.34	0.61	1.01	1.54	2.23	3.08	4.08		
Com	% Cites h	0.91	3.54	6.79	8.95	9.95	12.12	14.78	14.83	15.04	13.10	14.05	8.13
	Cum. % Cites H	0.01	0.04	0.11	0.20	0.30	0.42	0.57	0.72	0.87	1.00		
	Sum Cum. % Cites $\sum H$	0.01	0.05	0.17	0.37	0.67	1.09	1.66	2.38	3.25	4.25		
Econ	% Cites j	0.72	3.19	5.97	7.81	9.32	12.02	14.76	15.07	15.20	15.93	19.14	10.86
	Cum. % Cites J	0.01	0.04	0.10	0.18	0.27	0.39	0.54	0.69	0.84	1.00		
	Sum Cum. % Cites $\sum J$	0.01	0.05	0.15	0.32	0.59	0.98	1.52	2.21	3.05	4.05		
Ind Rel & Lab	% Cites k	1.16	4.01	7.32	8.01	8.63	11.53	14.32	15.63	15.17	14.22	14.31	8.26
	Cum. % Cites K	0.01	0.05	0.12	0.21	0.29	0.41	0.55	0.71	0.86	1.00		
	Sum Cum. % Cites $\sum K$	0.01	0.06	0.19	0.39	0.68	1.09	1.64	2.35	3.20	4.20		
Inf Sc & Lib Sc	% Cites m	1.64	4.58	7.43	9.28	10.06	12.79	13.87	13.87	13.29	13.19	15.02	8.87
	Cum. % Cites M	0.02	0.06	0.14	0.23	0.33	0.46	0.60	0.74	0.87	1.00		
	Sum Cum. % Cites $\sum M$	0.02	0.08	0.21	0.44	0.77	1.23	1.83	2.56	3.43	4.43		
Mgt	% Cites n	0.44	1.89	4.12	6.68	8.32	11.30	14.72	16.33	17.78	18.43	37.06	20.22
	Cum. % Cites N	0.00	0.02	0.06	0.13	0.21	0.33	0.47	0.64	0.82	1.00		
	Sum Cum. % Cites $\sum N$	0.00	0.03	0.09	0.22	0.44	0.77	1.24	1.88	2.69	3.69		
Average Percent Cites		0.88	3.25	6.05	7.95	9.16	12.05	14.60	15.37	15.47	15.23	22.01	

Table 5 shows the discount factors ($\sum_{i=1}^{n} e^{-ri} f_i$) for subjects at various discount rates. The magnitude of these is similar to those for economics journals above, and the differences in discount factors are similar with a maximum difference between the subjects Management (Mgt) and Information Science and Library Science (IS) relative to the undiscounted value of a cite of 4.5% at 10%, 5.7% at 20% and 5.3% at 40% . These translate to differences of 8.3%, 18.3% and 42.7% of the discounted value of cites. Thus for business school subjects, differences in citation timing have a significant impact on the estimated differences in contributions to knowledge when exponential discounting is used to determine the value of delay, particularly at high discount rates.

Table 5: Discount Factors over 10 Years for Business Subjects

	r=0.01	*r*=0.05	*r*=0.1	*r*=0.2	*r*=0.3	*r*=0.4
BUSINESS (B)	0.94	0.74	0.55	0.32	0.20	0.13
BUSINESS, FINANCE (F)	0.94	0.75	0.57	0.34	0.22	0.15
COMMUNICATION (C)	0.94	0.76	0.58	0.36	0.23	0.16
ECONOMICS (E)	0.94	0.75	0.57	0.34	0.22	0.15
INDUSTRIAL RELATIONS & LABOR (IR)	0.94	0.75	0.58	0.35	0.23	0.16
INFORMATION SCIENCE & LIBRARY SCIENCE (IS)	0.95	0.76	0.59	0.37	0.25	0.18
MANAGEMENT (M)	0.94	0.73	0.55	0.31	0.19	0.12

Example Three:
Comparisons of Citation Patterns for Leading Journals in Economics and Neurosciences

A number of papers have commented on differences in citation patterns across disciplines.[17] Anderson and Tressler (2016) use comparisons of the rate of citation capture to show that citations are received much earlier in leading journals in the Neurosciences than they are in Economics. Here we apply the techniques introduced in this paper to the data used in the previous paper to confirm the comparison and to indicate the possible importance of the differences using the exponential delay function.

Table 6 reports 10 year citation patterns for three leading Journals in Neuroscience, *Nature Reviews Neuroscience* (*NRN*), *Annual Review of Neuroscience* (*ARN*), *Nature Neuroscience* (*NN*), and three leading journals in Economics, *Journal of Economic Literature* (*JEL*), *Quarterly Journal of Economics* (*QJE*)and *Econometrica* (*EM*).[18] Again, papers eligible to receive citations in each discipline category are those published in the journals considered in 2003 while citations to these papers are from all journals in the WoK databases from 2003 to 2012. As indicated by a comparison of the cumulative citation patterns reported in Table 6, based only on the assumption that delay increases with time $d'(t) > 0$, publications in the neuroscience journals unambiguously involve less delay that those in economics in all pairwise comparisons.

[17] See, for example, Nederhof (2006), Evidence (2007), Levitt and Thelwall (2008) and THE (2011).

[18] Comparisons involving further journals are available in Anderson and Tressler (2016).

Table 6: Citation Patterns for Leading Economic and Neuroscience Journals 10YR ISI Cites to 2003 Publications

Name of Journal		**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**Cites/ Paper C**	**PV C r=0.1**
J ECON LIT	% Cites *f*	0.80	3.87	6.94	9.14	9.34	13.08	16.34	13.28	12.47	14.74	71.4	11.51
	Cum. % Cites F	0.0080	0.0467	0.1161	0.2075	0.3009	0.4316	0.5951	0.7278	0.8526	1.0000		
	Sum Cum. % Cites $\sum F$	0.0080	0.0547	0.1708	0.3783	0.6791	1.1107	1.7058	2.4336	3.2862	4.2862		
Q J ECON	% Cites *g*	0.45	2.25	5.06	7.00	8.24	12.08	14.25	15.58	17.37	17.70	121.0	15.83
	Cum. % Cites *G*	0.0045	0.0271	0.0777	0.1477	0.2301	0.3510	0.4935	0.6492	0.8230	1.0000		
	Sum Cum. % Cites $\sum G$	0.0045	0.0316	0.1093	0.2570	0.4871	0.8380	1.3315	1.9808	2.8038	3.8038		
ECONOMETRICA	% Cites *h*	0.75	2.64	5.39	6.79	9.07	11.12	14.46	16.25	15.65	17.88	63.3	8.70
	Cum. % Cites *H*	0.0075	0.0339	0.0878	0.1557	0.2464	0.3576	0.5022	0.6647	0.8212	1.0000		
	Sum Cum. % Cites $\sum H$	0.0075	0.0415	0.1293	0.2850	0.5315	0.8891	1.3913	2.0560	2.8772	3.8772		
NAT REV NEUROSCI	% Cites *j*	1.19	6.82	10.47	11.23	11.38	11.28	11.65	12.07	11.69	12.22	236.5	47.93
	Cum. % Cites *J*	0.0119	0.0800	0.1848	0.2971	0.4109	0.5237	0.6402	0.7609	0.8778	1.0000		
	Sum Cum. % Cites $\sum J$	0.0119	0.0919	0.2767	0.5738	0.9847	1.5083	2.1485	2.9094	3.7873	4.7873		
ANNU REV NEUROSCI	% Cites *k*	0.94	8.30	12.94	12.96	11.80	11.47	11.15	10.58	10.19	9.68	213.7	48.06
	Cum. % Cites *K*	0.0094	0.0924	0.2217	0.3513	0.4693	0.5840	0.6955	0.8013	0.9032	1.0000		
	Sum Cum. % Cites $\sum K$	0.0094	0.1017	0.3234	0.6747	1.1440	1.7280	2.4235	3.2248	4.1279	5.1279		
NAT NEUROSCI	% Cites *m*	2.14	8.65	11.14	11.45	11.03	10.93	11.34	11.13	11.29	10.89	167.9	37.88
	Cum. % Cites *M*	0.0214	0.1079	0.2193	0.3338	0.4441	0.5534	0.6669	0.7782	0.8911	1.0000		
	Sum Cum. % Cites $\sum M$	0.0214	0.1293	0.3487	0.6825	1.1267	1.6801	2.3470	3.1251	4.0162	5.0162		
Average Percent Cites		1.05	5.42	8.66	9.76	10.14	11.66	13.20	13.15	13.11	13.85		

Table 7 gives the discount factors for the six journals at different discount rates. When exponential discounting is used to indicate the magnitudes of the impact of time delay, the differences are significantly greater than those obtained in the comparisons considered above. As a percentage of the undiscounted value of a cite the maximum differences in the discount factor is between the *QJE* and *ARN* at 8.2% for a discount rate of 10% and 10.4% for a discount rate of 20%. These are 14.7% and 32.3% of the discounted value of a cite, respectively. At a discount rate of 40% the maximum difference in the undiscounted value of a cite is between the *QJE* and *NN* at 9.5%, which is 72.5% of the discounted value. In this case, the substantial differences between cites per paper mean that taking into account the cost of delay does not change the ranking of journals until discount rates exceed 37%, at which point the *NRN* and *ARN* switch.

Table 7: Discount Factors over 10 Years for Economics and Neuroscience Journals

	r=0.01	r=0.05	r=0.1	r=0.2	r=0.3	r=0.4
J ECON L (JEL)	0.94	0.76	0.58	0.36	0.23	0.16
Q J ECON (QJE)	0.94	0.74	0.55	0.32	0.20	0.13
ECONOMETICA (EM)	0.94	0.74	0.56	0.33	0.21	0.14
NAT REV NEUROSCI (NRN)	0.95	0.78	0.61	0.40	0.28	0.20
ANNU REV NEUROSCI (ARN)	0.95	0.79	0.63	0.43	0.30	0.22
NAT NEUROSCI (NN)	0.95	0.79	0.63	0.42	0.30	0.23

Example Four:
Comparisons of Citation Patterns Leading Individual Economists in New Zealand

In all of the examples above we have dealt with cases in which there were a relatively large number of papers to which citations could be attracted, all articles in a journal in a particular year or all articles in all journals in a subject area in a particular year. We now consider citations patterns for individuals using data collected for the research reported in Tressler and Anderson (2012). We have selected the top seven New Zealand economics in terms of the total number of WoS citations received over a ten year period to articles published in 2000 and 2001.[19] The seven economists are Philip McCann (*Mc*), Donggyu Sul (*Su*), John McDermott (*McD*), John Gibson (*Gi*), Dean Hyslop (*Hy*), Mark Holmes (*Ho*) and David Fielding (*Fi*). Since we are dealing with a relatively small number of papers for each individual, more variation in the data would be expected. The citations patterns for each of the seven individuals over the 10 years are described in Table 8 along with cumulative cites and the sums of cumulative cites.

[19] Cites are from WoS journals in economics using a definition of economics based on journals listed as economics journals in the Journal Citation Reports. The seven economists are those with the highest number of share-adjusted cites to articles published in 2000 and 2001 and collected over a 10 year- period commencing in the year of publication. Note that (a) the pool from which our seven economists were selected consisted of all full-time economists employed in New Zealand university-based economics departments on the 15th of April 2007 and/or the 15th of April 2009 and that (b) we have adopted the 1/*n* rule to allocate cites between authors of multi-authored papers, where n equals the number of authors on a paper.

Table 8: Citations Patterns Seven New Zealand Economists, 10YR ISI Cites to 2000 and 2001 Publications

Individual		**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**	**Year 7**	**Year 8**	**Year 9**	**Year 10**	**Total Cites C**	**PV C r=0.1**
McCann (*Mc*)	% Cites *f*	0.53	0.53	4.79	7.45	16.49	10.11	18.62	11.70	14.36	15.43	188.00	106.00
	Cum. % Cites F	0.0053	0.0106	0.0585	0.1330	0.2979	0.3989	0.5851	0.7021	0.8457	1.0000		
	Sum Cum. % Cites ∑*F*	0.0053	0.0160	0.0745	0.2074	0.5053	0.9043	1.4894	2.1915	3.0372	4.0372		
Sul (*Su*)	% Cites *g*	0.00	3.08	12.31	13.85	9.23	7.69	9.23	7.69	16.92	20.00	65.00	38.09
	Cum. % Cites *G*	0.0000	0.0308	0.1538	0.2923	0.3846	0.4615	0.5538	0.6308	0.8000	1.0000		
	Sum Cum. % Cites ∑*G*	0.0000	0.0308	0.1846	0.4769	0.8615	1.3231	1.8769	2.5077	3.3077	4.3077		
McDermott (*McD*)	% Cites *h*	0.00	0.00	2.33	20.93	9.30	9.30	9.30	23.26	9.30	16.28	43.00	24.40
	Cum. % Cites *H*	0.0000	0.0000	0.0233	0.2326	0.3256	0.4186	0.5116	0.7442	0.8372	1.0000		
	Sum Cum. % Cites ∑*H*	0.0000	0.0000	0.0233	0.2558	0.5814	1.0000	1.5116	2.2558	3.0930	4.0930		
Gibson (*Gi*)	% Cites *j*	2.63	2.63	7.89	15.79	7.89	2.63	10.53	21.05	13.16	15.79	38.0	22.28
	Cum. % Cites *J*	0.0263	0.0526	0.1316	0.2895	0.3684	0.3947	0.5000	0.7105	0.8421	1.0000		
	Sum Cum. % Cites ∑*J*	0.0263	0.0789	0.2105	0.5000	0.8684	1.2632	1.7632	2.4737	3.3158	4.3158		
Hyslop (*Hy*)	% Cites *k*	2.94	2.94	5.88	14.71	11.76	14.71	14.71	8.82	8.82	14.71	34.0	20.57
	Cum. % Cites *K*	0.0294	0.0588	0.1176	0.2647	0.3824	0.5294	0.6765	0.7647	0.8529	1.0000		
	Sum Cum. % Cites ∑*K*	0.0294	0.0882	0.2059	0.4706	0.8529	1.3824	2.0588	2.8235	3.6765	4.6765		
Holmes (*Ho*)	% Cites *m*	0.00	0.00	3.70	7.41	11.11	14.81	3.70	3.70	29.63	25.93	27.00	14.22
	Cum. % Cites *M*	0.0000	0.0000	0.0370	0.1111	0.2222	0.3704	0.4074	0.4444	0.7407	1.0000		
	Sum Cum. % Cites ∑*M*	0.0000	0.0000	0.0370	0.1481	0.3704	0.7407	1.1481	1.5926	2.3333	3.3333		
Fielding (*Fi*)	% Cites *n*	0.00	0.00	8.70	13.04	8.70	4.35	8.70	17.39	17.39	21.74	23.00	12.72
	Cum. % Cites *N*	0.0000	0.0000	0.0870	0.2174	0.3043	0.3478	0.4348	0.6087	0.7826	1.0000		
	Sum Cum. % Cites ∑*N*	0.0000	0.0000	0.0870	0.3043	0.6087	0.9565	1.3913	2.0000	2.7826	3.7826		
Average % Cites		0.87	1.31	6.51	13.31	10.64	9.09	10.68	13.37	15.66	18.55		

For any delay function with $d'(t) > 0$, the cost of the delay in the uptake of knowledge is unambiguously greater for: *Mc* compared with *Hy*, *Ho* compared with *Mc*, *Ho* compared with *Su*, *Fi* compared with *Su*, *McD* compared with *Hy*, *Ho* compared with *Gi*, *Fi* compared with *Gi*, *Ho* compared with *Hy* and *Fi* compared with *Hy*. For a delay function with $d'(t) > 0$ and $d''(t) < 0$, in addition to these comparisons delay is unambiguously greater for *Mc* compared with *Gi*, *McD* compared with *Su*, *McD* compared with *Gi* and *Ho* compared with Fi. Using the notation above it follows that $Ho \succcurlyeq_d^1 Mc \gtrsim_d^1 Hy$, $Ho \succcurlyeq_d^1 Mc \gtrsim_d^2 Gi$ and $Ho \succcurlyeq_d^2 Fi \gtrsim_d^1 Su$. Overall, while comparisons of a number of pairs are possible in terms of the cost of delay, there are only a small number of three person comparison chains.

Table 9 shows the discount factors ($\sum_{i=1}^{n} e^{-ri} f_i$) for a range of discount rates. For all discount rates shown under 40%, the rank of individuals from the highest discount factor (lowest delay) to the lowest discount factor remains the same: *Hy*, *Gi*, *Sul*, *McD*, *Mc*, *Fi* and *Ho*.[20] For individuals, given the exponential delay function, the cost of time delays as a percentage of the undiscounted value of a cite differs by a maximum of 7.8% at a 10% discount rate for *Hy* compared with *Ho*, rising to 9.6% at a discount rate of 20% and 8.4% at 40%. These differences represent 14.9%, 32.7% and 78.7% of the value of a discounted cite at these discount rates. Thus, the magnitude of the impact of differences in the timing of citations on estimates of the value of knowledge contributions can be very significant.

Table 9: Discount Factors over 10 Years for Individual Economists

	r=0.01	*r*=0.05	*r*=0.1	*r*=0.2	*r*=0.3	*r*=0.4
McCann (*Mc*)	0.94	0.75	0.56	0.33	0.21	0.13
Sul (*Su*)	0.94	0.76	0.59	0.37	0.25	0.17
McDermott (*McD*)	0.94	0.75	0.57	0.34	0.21	0.14
Gibson (*Gi*)	0.95	0.76	0.59	0.37	0.25	0.18
Hyslop (*Hy*)	0.95	0.77	0.61	0.39	0.26	0.19
Holmes (*Ho*)	0.94	0.72	0.53	0.29	0.17	0.11
Fielding (*Fi*)	0.94	0.74	0.55	0.33	0.20	0.13

As shown in Table 9, when the seven economists are ranked by total citations received over the 10 year period, the ranking is *McC*, *Sul*, *McD*,*Gi*, *Hy*, *Ho* and *Fi*. When the cost of time delay is taken into account, this ordering changes at interest rates over 24%. Between 24% and 25% *McD* and *Gi* change order, between 29% and 30% *Ho* and *Fi* change order and between 30 and 31% *McD* and *Hy* change order. Thus, for these economists, taking into account the timing of the flow of citations would affect the ranking of the economists by research productivity, but only for relatively high discount rates.[21]

[20] There are some changes in discount rates for discount rates over 40%.

[21] These discount rates are not high compared with the implicit discount rates used by RePEc.

4. Conclusions

Research publications not only attract different numbers of citations, but also have different citation patterns over time. Viewing citations as indicators of contributions to knowledge, time delays in the receipt of cites suggest that knowledge diffusion is slower. For a given number of citations received, this suggests that when citations are slower coming in the impact of research may be less.

In this paper we have provided a framework for comparing different time patterns of citations. Using this framework we have shown that the contribution to knowledge can be treated as depending on the number of citations and a function representing the delay in the uptake of knowledge. A simple approach drawn from the stochastic dominance literature was then used to show that comparisons can be made based only on the assumption that cost of delay increases with time, or that the cost of delay increases with time and at a decreasing rate. Generally these comparisons only provide a partial ordering of the citation patterns being compared. If a particular delay function is assumed then a complete ordering is obtained and the magnitude of differences determined. As an example we have used a delay function that leads to exponential discounting.

We have applied this framework to a series of ten year citation patterns drawn from existing research for: different economics journals, different subject areas with business, leading journals in economics compared with neuroscience and publications by individual economists. For data from publications in different business subjects an almost complete order based on the cost of delay is possible with knowledge diffusion from publications in Management being the slowest and Information Science and Library Science the fastest. Given the assumption that the cost of delay increases with time, our data also suggests that based on citations received by leading journals in Economics and Neuroscience, knowledge diffusion is unambiguously slower in Economics. However, in the other examples presented, only comparisons of pairs are generally possible without additional restrictions on the nature of the cost of delay.

When a delay function consistent with exponential discounting is used we have been able to show how the cost of delays in knowledge diffusion vary with the assumed discount rate. We show that differences in value resulting from difference in citations time patterns can be quite high, particularly for high discount rates. In the examples provided the differences were particularly large when comparing citation time patterns for leading journals in Economics compared with Neuroscience and for comparisons of individual economists. Even so, the overall valuation of the contribution to knowledge was generally dominated by the number of citations received, with relatively few changes in rankings resulting from a consideration of citation timing.

There are clearly many important questions that remain in assessing the impact of citation timing. In this paper we have assumed that when citations are used as indicators of contributions to knowledge made in a research publication, the contribution signalled by that citation does not depend on the number of citations occurring at the same time or the number that have been received previously. While this is consistent with the way in which citations are typically used in research evaluation, it is clearly a simplification. A more general approach would be to develop a formal model of knowledge diffusion. Such an approach might also be able to provide a more complete characterisation of appropriate restrictions on delay functions and a basis for considering approaches to discounting.

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