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A Stochastic Sub-national Population Projection Methodology with an Application to the Waikato Region of New Zealand

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Abstract
In this paper we use a stochastic population projection methodology at the sub-national level as an alternative to the conventional deterministic cohort-component method. We briefly evaluate the accuracy of previous deterministic projections and find that there is a tendency for these to be conservative: under-projecting fast growing populations and over-projecting slow growing ones. We generate probabilistic population projections for five demographically distinct administrative areas within the Waikato region of New Zealand, namely Hamilton City, Franklin District, Thames-Coromandel District, Otorohanga District and South Waikato District. Although spatial interaction between the areas is not taken into account in the current version of the methodology, a consistent set of cross-regional assumptions is used. The results are compared to official sub-national deterministic projections. The accuracy of sub-national population projections is in New Zealand strongly affected by the instability of migration as a component of population change. Unlike the standard cohort-component methodology, in which net migration levels are projected, the key parameters of our stochastic methodology are age-gender-area specific net migration rates. The projected range of rates of population growth is wider for smaller regions and/or regions more strongly affected by net migration. Generally, the identified and modelled uncertainty makes the traditional 'mid range' scenario of sub-national population projections of limited use for policy analysis or planning beyond a relatively short projection horizon. Directions for further development of a stochastic sub-national projection methodology are suggested.

Keywords: cohort-component model, stochastic simulation, population, fertility, mortality, migration, sub-national area

JEL Classification: J11, R23

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1 Introduction

Cross-border population mobility is globally increasing in intensity, complexity and volatility (e.g., Poot *et al.*, 2008). National population projections can account for growing uncertainty in international migration by widening the considered range of migration levels or by adopting a probabilistic framework. However, international migration is strongly spatially selective, often affecting some regions (predominantly metropolitan areas) more than others (provincial and rural areas). Sub-national population projections therefore ideally combine spatially-varying degrees of uncertainty with respect to international migration, with endogenous – but relatively more stable – patterns of internal population redistribution. In this paper we use a stochastic population projections methodology at the regional level as an alternative to the conventional deterministic cohort-component method.

Due to its transparency and simplicity, the cohort-component method of projecting future populations is still commonly used by forecasting agencies in many countries, particularly at the sub-national level. The assumptions used in such projection exercises can be derived from available statistical information, but they may also be informed by local-level expert consultation (e.g., Cameron *et al.*, 2007; 2008c). With this methodology the variability of future demographic drivers of population change is signalled by the formulation of a range of scenarios that either yield relatively high or relatively low future population levels. Hybrid methods already exist that combine cohort-component techniques of national population projections with econometric modelling of volatile international migration, at least for relatively short projection horizons (e.g., Gorbey *et al.*, 1999). However, deterministic projection exercises require policymakers to gauge population trends by means of a limited number of assumed scenarios and may therefore be misleading (Bryant, 2005). For example, the reported projections could give the impression that the future population trajectory must remain within the calculated upper and lower bounds.

Stochastic population projection techniques that allow for probabilistic formulation of the parameters of the underlying demographic processes are increasingly important as the observed variability about such processes increases spatially and temporally, but also when the parameters are likely to be correlated. It should be stated at the outset that we make no attempt in this paper to generate a full set of “bottom up” sub-national stochastic projections that aggregate to a projected national population with plausible net international migration outcomes and net internal migration summing to zero. The choice of the appropriate model for the matrix of gross internal migration flows is at the core of multiregional demography (e.g. Rogers, 1995) but is still a considerable methodological (and even computational) challenge in a stochastic framework. Statistics New Zealand (2008, p.27) go as far as to conclude that “Applying a stochastic approach to sub-national population projections... may be unattainable”. However, a probabilistic approach has already been applied successfully at the regional level in a number of countries, reviewed by Wilson and Bell (2004), who themselves then adopt such an approach for projecting the population of Queensland in Australia.[1]

[1] The studies mentioned by Wilson and Rees (2004) include applications to NUTS2 regions in Europe and California in the USA.

In this paper we briefly evaluate the accuracy of the conventional cohort-component projection methodology for regional population projections before outlining and applying a stochastic projection methodology for sub-regions of the Waikato region. We assess the accuracy of cohort-component projections by comparing past regional Statistics New Zealand (SNZ) projections with actual outcomes. A more detailed assessment of accuracy was conducted by Statistics New Zealand (2008) itself. SNZ find that projections become less accurate for smaller geographical areas and when projecting further into the future. The relative error is quite small with a short horizon of three years but becomes rather large with a 13 year horizon.[2] In our own assessment of the accuracy of past SNZ projections, we find that the medium series (which are often seen as the preferred ones) are quite good at predicting the ranking of sub-national areas in terms of population growth rates, but less good at predicting the levels of population growth (Cameron *et al.*, 2008a). Fast-growing areas are systematically under-projected, while areas with slow or negative growth are systematically over-projected. This built-in conservatism is consistent with what Statistics New Zealand (2008) concludes itself on the reliability of their sub-national projections.

We therefore offer a way forward and suggest that stochastic projections may reduce the likelihood of this systematic under- or over-projecting of population growth. Like deterministic projections, stochastic projections will show greater uncertainty further into the future and for smaller geographical areas, but in the stochastic projections this uncertainty is explicitly modelled so that the likelihood of various ranges of outcomes can be quantified. This makes it also easier to explain the results to laypeople (Bryant, 2005). To demonstrate the methodology, we generate probabilistic population projections for five administrative areas within the Waikato region of New Zealand, namely Hamilton City, Franklin District, Thames-Coromandel District, Otorohanga District and South Waikato District. These regions were selected because they are included in a recently developed integrated land use model of the Waikato region that has demographic, economic and environmental dimensions (see Huser *et al.*, 2009; Rutledge *et al.*, 2009), but also because they represent a range of different types of regions varying from the urban core (Hamilton City) to the rural periphery (Otorohanga District). Although spatial interaction between the areas is not taken into account in the current version of the methodology, a consistent set of cross-regional assumptions is used.[3] The results are compared to the official sub-national deterministic projections published by Statistics New Zealand (2007).[4] Of course, the extent to which the median value of the distribution of population outcomes generated by the stochastic methodology is more or less accurate than the "preferred" (medium)

[2] See for example Statistics New Zealand (2008), Table 8. Using a 1993 released projection, it is found that the difference between the medium projected and actual population is less than 5 percent for 60 out of 73 Territorial Local Authorities (TLAs) when projecting three years out to 1996. However, projecting 13 years out to 2006 reduces the number of TLAs for which the relative projection error is less than 5 percent to 26 out of 73.

[3] However, no attempt is made to generate a set of projections that aggregate up to a set of national projections. The objective here is to show the differences in applying the methodology to widely varying regions from urban and peri-urban through to peripheral and rural.

[4] Since completion of this paper, an updated set of subnational population projections has been released (Statistics New Zealand, 2010). The new projections suggest slightly faster population growth for the Waikato region than the previous set of projections (Statistics New Zealand, 2007). The differences are actually very small (about 2 percent in the medium projection for 2031) and have a negligible impact on the comparison of deterministic and stochastic projections given in Section 6 of the present paper.

projection generated by the conventional methodology cannot be assessed for some time to come, but there are a number of other criteria by which population projection methodologies can be assessed (such as coherence, interpretability and policy relevance). Based on such criteria stochastic projections offer much promise (e.g. Bryant, 2005; Wilson, 2005) and probabilistic methodologies have now been used for a large number of countries (see, e.g., Wilson and Rees, 2005), although applications to multiple sub-national areas – such as reported in the present paper – are still quite rare. In the final section of the paper we comment on possible future developments of the stochastic population projection methodology at the sub-national level.

2 The Deterministic Cohort-Component Method

The deterministic and stochastic cohort-component models for population projections have in common a stock-flow description of population dynamics that is based on the following fundamental 'accounting identity':

The population usually resident in area *l* at the end of year *t*
= The population usually resident in area *l* at the beginning of year *t*
+ births to mothers residing in area *l* during year *t*
– deaths of residents of area *l* during year *t*
+ inward migration from other regions and from overseas into region *l* during year *t*
– outward migration of residents from area *l* to other regions or to overseas during year *t*

Starting with a given base year population, the population twelve months later is then calculated with the equation above through modelling births, deaths and migration. The projected population is then used as the base to calculate the population of the following year. This procedure is repeated for each year through to the end of the projection period, and separately for each gender. To calculate projected births, deaths, inward and outward migration, the cohort-component model uses the age and sex composition of the projected population in each year. Thus, the actual calculations are done for males and females separately in each age group (single year or five-year age group).

Sets of assumptions are used for each of the demographic 'drivers'. In the case of New Zealand's official population projections produced by SNZ, the fertility assumptions are derived from total and age-specific fertility rates, which are derived from recent observed trends in births by age of the mother. The assumed age-specific fertility rates are multiplied by the numbers of women in the corresponding age group in which childbearing age takes place (13-49) to determine the number of births in each year. The SNZ mortality assumptions are derived from trends in age-sex specific deaths and the estimated corresponding life expectancy at birth, again using recent historical trends. The assumed age- and gender-specific mortality (or their complement, survivorship) rates are multiplied by the numbers of people of each age and gender to determine the expected number of deaths of each age and gender in each year. So population change is by this method in part determined by the perfectly predictable ageing of the population (a person aged *a* in year *t* will be aged *a*+1 in year *t*+1), and in part by the relatively slowly changing fertility and mortality trends.

Furthermore, inward migration and outward migration are not considered separately in the conventional methodology – instead an assumption is made about the level of net migration (in minus out). This is often done for convenience rather than theoretical appeal, as a 'net migrant' does not exist (Rogers, 1990). Net migration is much less stable or predictable than natural increase, and although the net flows may be small as a proportion of the total population in some regions, the effect of differences in net migration assumptions can have a large effect on the projected population at longer projection horizons. The SNZ net migration assumptions are based on historical trends in net migration, supplemented by additional data such as building consents and international arrivals data. A key feature is that these assumptions are constrained such that the total net migration across all sub-national population projections must sum to the total net migration for the country in the national population projections. This means that if total net migration into New Zealand is underestimated or overestimated, then it is likely that the sub-national population projections will all be affected (Statistics New Zealand, 2008).

Finally, it is unrealistic to expect population projections to be 'correct', as the assumptions will never be exactly realised. As such, there will almost always be an observable difference between the actual and projected populations, which reflects the difference between actual and assumed fertility, mortality, and net migration. The following section looks at these observed differences for past SNZ population projections.

3 The Accuracy of Past SNZ Projections

Table 1 compares the SNZ 1991-base medium variant population projections for each region for 2006 with the actual 2006 (March) Census usually resident populations (i.e. that exclude residents temporarily overseas). The SNZ 1991-base medium variant population projections were published in January 1993. Columns (1) and (2) show the actual Census night usually resident populations in 1991 and 2006 respectively. Column (3) shows the SNZ projected population for 2006 under their medium assumptions. Columns (4) and (5) show the actual and projected population growth rates between 1991 and 2006, and Column (6) shows the difference between the two. Column (7) shows the ranking of the sixteen regions in terms of the actual rates of population growth, and Column (8) shows the projected ranking.

As Table 1 shows, actual population growth was positive in all but two regions over the period 1991 to 2006, and was highest in the Tasman (42.1 percent), Auckland (40.0 percent), and Marlborough (31.5 percent) regions.[5] In the SNZ 1991-base medium projection, the projected population growth was lower than actual population growth in 14 of the 16 regions (Taranaki and Manawatu/Wanganui being the exceptions) over the period to 2006, and the difference between actual and projected population growth was greatest in the three regions with the highest population growth.

[5] If Tasman and Nelson, both regions with small populations would have been combined, Auckland would have been the fastest growing region since 1991. It should be noted that net international migration into Auckland and natural increase are jointly responsible for the growth in the Auckland region, not net internal migration (see Poot, 2005, Table 3.1).

Table 1: Comparison of SNZ 1991-base population projections published in January 1993 with the actual 2006 population

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			Medium		**1991-base**			
	Actual CURP	**Actual CURP**	**Projected URP**	**Actual Pop Growth**	**Proj. Pop Growth**	**Actual-Proj.**	**Actual Growth Rank**	**Projected Growth Rank**
	1991	**2006**	**2006**	**1991-2006**	**1991-2006**	**1991-2006**	**1991-2006**	**1991-2006**
Northland	126900	154392	141700	21.7%	11.7%	10.0%	7	8
Auckland	943620	1321074	1170900	40.0%	24.1%	15.9%	2	1
Waikato	331230	393171	377300	18.7%	13.9%	4.8%	8	6
Bay of Plenty	203898	264180	250100	29.6%	22.7%	6.9%	4	2
Gisborne	44262	48681	42500	10.0%	-4.0%	14.0%	12	15
Hawkes Bay	138333	151755	141900	9.7%	2.6%	7.1%	13	13
Taranaki	107175	104697	114000	-2.3%	6.4%	-8.7%	15	12
Manawatu-Wanganui	224793	225696	245100	0.4%	9.0%	-8.6%	14	11
Wellington	400275	456654	438800	14.1%	9.6%	4.5%	10	10
Tasman	34002	48306	39900	42.1%	17.3%	24.7%	1	4
Nelson	36465	45372	42800	24.4%	17.4%	7.1%	5	3
Marlborough	35130	46179	40900	31.5%	16.4%	15.0%	3	5
West Coast	31590	35844	31800	13.5%	0.7%	12.8%	11	14
Canterbury	437958	541515	495400	23.6%	13.1%	10.5%	6	7
Otago	177585	209850	196800	18.2%	10.8%	7.3%	9	9
Southland	99981	95247	93400	-4.7%	-6.6%	1.8%	16	16
New Zealand	3375188	4144619	3865306	22.8%	14.5%	8.3%	N/A	N/A

Note: (C)URP stands for (Census) Usually Resident Population

Source: Statistics New Zealand data and Population Studies Centre calculations.

Nonetheless, the projected ranking of the regions in terms of population growth rates was very close to the actual ranking that was obtained. So, in the SNZ 1991-base medium population projections, projected population growth rates were lower than actual population growth rates generally over the period 1991 to 2006, but the relative population growth rates between regions were quite close to those that actually obtained, i.e. the regions projected to grow the fastest actually grew the fastest. This conclusion reinforces the results of a similar analysis done by Poot (2005).

Table 2 compares the SNZ 1996-base medium variant population projections for each territorial and local authority (TLA) for 2006, published in October 1997, with the actual 2006 populations. The 1996 and 2006 populations shown in Columns (1) and (2) respectively are estimated usually resident populations at 30 June (the date that these projections are calculated for). Columns (3) through (8) have similar interpretations as for Table 1.

As Table 2 shows, population growth over the period 1996 to 2006 was positive in most TLAs and highest in Queenstown-Lakes District (62.8 percent) and Selwyn District (37.8 percent). Given that we focus in the paper specifically on the TLAs Hamilton City, Franklin District, Thames-Coromandel District, Otorohanga District and South Waikato District, we note that Franklin District (24.5 percent), and Hamilton City (21.0 percent) were among the top quarter of TLAs in terms of the rate of population growth over that period. South Waikato District (-9.7 percent) and Otorohanga District (-6.0 percent) were among the bottom quarter. In the SNZ 1996-base medium projection, the projected population growth was lower than actual population growth in 45 of the 73 TLAs and, with the exception of Central Otago District, the difference between actual and projected population growth was greatest in the TLAs with the highest population growth. The population growth rate in Hamilton City was higher than projected by 8.8 percentage points and in Franklin District by 3.9 percentage points. The population growth rate in Thames-Coromandel District was lower than projected by 12.3 percentage points and in Otorohanga District by 10.0 percentage points. Again, despite these differences and with some exceptions, the projected ranking of the 73 TLAs in terms of population growth rates was quite close to the actual ranking that was obtained. The correlation coefficient between the actual and projected rankings is 0.73.

Table 3 presents similar data to Tables 1 and 2, but for the SNZ 2001-base population projections for each TLA, which were published in November 2002. The table presents comparative data for both the medium (Column (3) to (8)) and high (Columns (9) to (12)) variants of the SNZ projections.

Table 2: Comparison of SNZ 1996-base population projections published in October 1997 with actual 2006 population

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			Medium		**1996-base**			
	Est. URP 30 June	**Est. URP 30 June**	**Projected URP**	**Actual Pop Growth**	**Proj. Pop Growth**	**Actual-Proj.**	**Actual Rank**	**Projected Rank**
	1996	**2006**	**2006**	**1996-2006**	**1996-2006**	**1996-2006**	**1996-2006**	**1996-2006**
Far North	54100	57500	61800	6.3%	14.2%	-7.9%	28	13
Whangarei	68000	76500	74100	12.5%	9.0%	3.5%	18	22
Kaipara	17700	18550	17800	4.8%	0.6%	4.2%	34	40
Rodney	68000	92400	88100	35.9%	29.6%	6.3%	3	2
North Shore City	178400	216900	209100	21.6%	17.2%	4.4%	9	10
Waitakere City	160200	195300	188000	21.9%	17.4%	4.6%	8	9
Auckland City	361900	428300	418900	18.3%	15.8%	2.6%	14	12
Manukau City	264200	347100	310900	31.4%	17.7%	13.7%	6	8
Papakura	40800	46900	45200	15.0%	10.8%	4.2%	16	19
Franklin	48900	60900	59000	24.5%	20.7%	3.9%	7	4
Thames-Coromandel	25200	26700	29800	6.0%	18.3%	-12.3%	29	7
Hauraki	17700	17600	18000	-0.6%	1.7%	-2.3%	50	37
Waikato	40000	45400	41900	13.5%	4.8%	8.7%	17	30
Matamata-Piako	30200	31200	29100	3.3%	-3.6%	7.0%	37	55
Hamilton City	111100	134400	124600	21.0%	12.2%	8.8%	11	18
Waipa	39600	43700	43000	10.4%	8.6%	1.8%	23	24
Otorohanga	9900	9310	10300	-6.0%	4.0%	-10.0%	60	31
South Waikato	25700	23200	23400	-9.7%	-8.9%	-0.8%	69	72
Waitomo	10000	9680	9100	-3.2%	-9.0%	5.8%	53	73
Taupo	31400	33400	34200	6.4%	8.9%	-2.5%	27	23
Western Bay of Plenty	35700	43300	42600	21.3%	19.3%	2.0%	10	5
Tauranga City	79200	106700	95600	34.7%	20.7%	14.0%	4	3
Rotorua	66100	68100	70100	3.0%	6.1%	-3.0%	39	27
Whakatane	33900	34500	34700	1.8%	2.4%	-0.6%	42	35
Kawerau	8100	7150	7600	-11.7%	-6.2%	-5.6%	70	64

Table 2 ctd.: Comparison of SNZ 1996-base population projections published in October 1997 with actual 2006 population

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			Medium		**1996-base**			
	Est. URP 30 June	**Est. URP 30 June**	**Projected URP**	**Actual Pop Growth**	**Proj Pop Growth**	**Actual-Proj**	**Actual Rank**	**Projected Rank**
	1996	**2006**	**2006**	**1996-2006**	**1996-2006**	**1996-2006**	**1996-2006**	**1996-2006**
Opotiki	9600	9200	10900	-4.2%	13.5%	-17.7%	54	14
Gisborne	46800	45900	46600	-1.9%	-0.4%	-1.5%	51	43
Wairoa	10100	8720	9500	-13.7%	-5.9%	-7.7%	72	63
Hastings	67700	73200	69900	8.1%	3.2%	4.9%	25	33
Napier City	54600	56800	56400	4.0%	3.3%	0.7%	35	32
Central Hawkes Bay	13300	13250	12900	-0.4%	-3.0%	2.6%	49	52
New Plymouth	69400	71100	68600	2.4%	-1.2%	3.6%	41	47
Stratford	9700	9120	9100	-6.0%	-6.2%	0.2%	61	65
South Taranaki	29600	27200	27500	-8.1%	-7.1%	-1.0%	66	66
Ruapehu	17200	14050	16400	-18.3%	-4.7%	-13.7%	73	60
Wanganui	45800	43800	45400	-4.4%	-0.9%	-3.5%	55	46
Rangitikei	16700	15150	15400	-9.3%	-7.8%	-1.5%	68	70
Manawatu	28600	29000	30200	1.4%	5.6%	-4.2%	44	29
Palmerston North City	74900	78500	80300	4.8%	7.2%	-2.4%	33	25
Tararua	19400	18050	18500	-7.0%	-4.6%	-2.3%	63	59
Horowhenua	30700	30600	30800	-0.3%	0.3%	-0.7%	48	41
Kapiti Coast	39300	47500	44200	20.9%	12.5%	8.4%	12	16
Porirua City	47800	50600	47500	5.9%	-0.6%	6.5%	31	45
Upper Hutt City	37500	39700	36000	5.9%	-4.0%	9.9%	30	56
Lower Hutt City	98300	101300	97700	3.1%	-0.6%	3.7%	38	44
Wellington City	162700	187700	172500	15.4%	6.0%	9.3%	15	28
Masterton	23200	23200	22400	0.0%	-3.4%	3.4%	47	54
Carterton	6900	7260	6600	5.2%	-4.3%	9.6%	32	58
South Wairarapa	9100	9120	8900	0.2%	-2.2%	2.4%	46	49

Table 2 ctd.: Comparison of SNZ 1996-base population projections published in October 1997 with actual 2006 population

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			Medium		**1996-base**			
	Est. URP 30 June	**Est. URP 30 June**	**Projected URP**	**Actual Pop Growth**	**Proj Pop Growth**	**Actual-Proj**	**Actual Rank**	**Projected Rank**
	1996	**2006**	**2006**	**1996-2006**	**1996-2006**	**1996-2006**	**1996-2006**	**1996-2006**
Tasman	38600	45800	43700	18.7%	13.2%	5.4%	13	15
Nelson City	41000	44300	46000	8.0%	12.2%	-4.1%	26	17
Marlborough	39000	43600	42700	11.8%	9.5%	2.3%	21	21
Kaikoura	3600	3730	3500	3.6%	-2.8%	6.4%	36	50
Buller	10700	9940	10400	-7.1%	-2.8%	-4.3%	64	51
Grey	13900	13550	14000	-2.5%	0.7%	-3.2%	52	39
Westland	8400	8620	8500	2.6%	1.2%	1.4%	40	38
Hurunui	9600	10750	9800	12.0%	2.1%	9.9%	20	36
Waimakariri	32900	44100	39100	34.0%	18.8%	15.2%	5	6
Christchurch (incl. BP)	324400	361800	347600	11.5%	7.2%	4.4%	22	26
Selwyn	25400	35000	29700	37.8%	16.9%	20.9%	2	11
Ashburton	25600	28000	25100	9.4%	-2.0%	11.3%	24	48
Timaru	43300	43800	41500	1.2%	-4.2%	5.3%	45	57
Mackenzie	4200	3900	4200	-7.1%	0.0%	-7.1%	65	42
Waimate	7800	7380	7200	-5.4%	-7.7%	2.3%	57	69
Chatham Islands	750	650	830	-13.3%	10.7%	-24.0%	71	20
Waitaki	21900	20700	20600	-5.5%	-5.9%	0.5%	58	62
Central Otago	15200	17050	14400	12.2%	-5.3%	17.4%	19	61
Queenstown-Lakes	14800	24100	19600	62.8%	32.4%	30.4%	1	1
Dunedin City	120400	122300	123300	1.6%	2.4%	-0.8%	43	34
Clutha	18300	17200	17700	-6.0%	-3.3%	-2.7%	62	53
Southland	31000	29200	28800	-5.8%	-7.1%	1.3%	59	67
Gore	13500	12400	12400	-8.1%	-8.1%	0.0%	67	71
Invercargill City	54000	51600	50000	-4.4%	-7.4%	3.0%	56	68

Source: Statistics New Zealand data and Population Studies Centre calculations.

Table 3: Comparison of SNZ 2001-base population projections published in November 2002 with actual 2006 population

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
			Med.		Med.			Med.	High	High		High
	EURP 30 June	EURP 30 June	Proj. URP	Actual Pop Growth	Proj. Pop Growth	Actual-Proj.	Actual Rank	Proj. Rank	Proj. URP	Proj. Pop Growth	Actual-Proj.	Proj. Rank
	2001	2006	2006	2001-06	2001-06	2001-06	2001-06	2001-06	2006	2001-06	2001-06	2001-06
Far North	56400	57500	58800	2.0%	4.3%	-2.3%	48	22	60100	6.6%	-4.6%	22
Whangarei	70000	76500	72100	9.3%	3.0%	6.3%	17	29	73500	5.0%	4.3%	28
Kaipara	18000	18550	17900	3.1%	-0.6%	3.6%	34	44	18300	1.7%	1.4%	43
Rodney	78500	92400	90600	17.7%	15.4%	2.3%	3	3	93500	19.1%	-1.4%	3
North Shore City	194200	216900	216000	11.7%	11.2%	0.5%	10	8	219500	13.0%	-1.3%	10
Waitakere City	176200	195300	195600	10.8%	11.0%	-0.2%	12	9	199100	13.0%	-2.2%	11
Auckland City	388800	428300	437900	10.2%	12.6%	-2.5%	14	6	445400	14.6%	-4.4%	5
Manukau City	298200	347100	333400	16.4%	11.8%	4.6%	4	7	340100	14.1%	2.3%	7
Papakura	42300	46900	44400	10.9%	5.0%	5.9%	11	20	45200	6.9%	4.0%	21
Franklin	53300	60900	57900	14.3%	8.6%	5.6%	8	14	59100	10.9%	3.4%	13
Thames-Coromandel	25800	26700	27300	3.5%	5.8%	-2.3%	32	18	27900	8.1%	-4.7%	16
Hauraki	17200	17600	16900	2.3%	-1.7%	4.1%	45	54	17200	0.0%	2.3%	54
Waikato	41300	45400	42400	9.9%	2.7%	7.3%	15	31	43100	4.4%	5.6%	32
Matamata-Piako	30300	31200	29900	3.0%	-1.3%	4.3%	35	50	30500	0.7%	2.3%	51
Hamilton City	119500	134400	129200	12.5%	8.1%	4.4%	9	15	131500	10.0%	2.4%	15
Waipa	41400	43700	43000	5.6%	3.9%	1.7%	25	24	43900	6.0%	-0.5%	24
Otorohanga	9600	9310	9400	-3.0%	-2.1%	-0.9%	67	56	9600	0.0%	-3.0%	54
South Waikato	24200	23200	23300	-4.1%	-3.7%	-0.4%	69	64	23900	-1.2%	-2.9%	64
Waitomo	9800	9680	9600	-1.2%	-2.0%	0.8%	60	55	9800	0.0%	-1.2%	54
Taupo	32500	33400	33500	2.8%	3.1%	-0.3%	38	28	34100	4.9%	-2.2%	29
Western Bay of Plenty	39300	43300	43100	10.2%	9.7%	0.5%	13	12	44200	12.5%	-2.3%	12
Tauranga City	93300	106700	107900	14.4%	15.6%	-1.3%	7	2	111400	19.4%	-5.0%	2
Rotorua	66900	68100	68700	1.8%	2.7%	-0.9%	49	30	70100	4.8%	-3.0%	30
Whakatane	34000	34500	34200	1.5%	0.6%	0.9%	50	35	34800	2.4%	-0.9%	38
Kawerau	7300	7150	6900	-2.1%	-5.5%	3.4%	63	73	7000	-4.1%	2.1%	73

Table 3 ctd.: **Comparison of SNZ 2001-base population projections published in November 2002 with actual 2006 population**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
			Med.		**Med.**			**Med.**	**High**	**High**		**High**
	EURP 30 June	**EURP 30 June**	**Proj. URP**	**Actual Pop Growth**	**Proj. Pop Growth**	**Actual-Proj.**	**Actual Rank**	**Proj. Rank**	**Proj. URP**	**Proj. Pop Growth**	**Actual-Proj.**	**Proj. Rank**
	2001	**2006**	**2006**	**2001-06**	**2001-06**	**2001-06**	**2001-06**	**2001-06**	**2006**	**2001-06**	**2001-06**	**2001-06**
Opotiki	9500	9200	9800	-3.2%	3.2%	-6.3%	68	27	10100	6.3%	-9.5%	23
Gisborne	45500	45900	45000	0.9%	-1.1%	2.0%	53	47	46300	1.8%	-0.9%	42
Wairoa	9300	8720	8900	-6.2%	-4.3%	-1.9%	71	70	9100	-2.2%	-4.1%	69
Hastings	69600	73200	70400	5.2%	1.1%	4.0%	27	34	71700	3.0%	2.2%	34
Napier City	55200	56800	55500	2.9%	0.5%	2.4%	36	36	56800	2.9%	0.0%	35
Central Hawkes Bay	13200	13250	13000	0.4%	-1.5%	1.9%	55	53	13300	0.8%	-0.4%	50
New Plymouth	68400	71100	68000	3.9%	-0.6%	4.5%	29	45	69400	1.5%	2.5%	46
Stratford	9100	9120	8700	0.2%	-4.4%	4.6%	56	71	8900	-2.2%	2.4%	70
South Taranaki	28400	27200	27500	-4.2%	-3.2%	-1.1%	70	62	28100	-1.1%	-3.2%	62
Ruapehu	15000	14050	14400	-6.3%	-4.0%	-2.3%	72	67	14800	-1.3%	-5.0%	65
Wanganui	44400	43800	43900	-1.4%	-1.1%	-0.2%	61	48	45100	1.6%	-2.9%	45
Rangitikei	15500	15150	14800	-2.3%	-4.5%	2.3%	64	72	15200	-1.9%	-0.3%	67
Manawatu	28200	29000	28800	2.8%	2.1%	0.7%	37	33	29300	3.9%	-1.1%	33
Palmerston North City	75200	78500	78700	4.4%	4.7%	-0.3%	28	21	80600	7.2%	-2.8%	20
Tararua	18300	18050	17700	-1.4%	-3.3%	1.9%	62	63	18100	-1.1%	-0.3%	63
Horowhenua	30600	30600	30500	0.0%	-0.3%	0.3%	57	43	31100	1.6%	-1.6%	44
Kapiti Coast	43600	47500	48100	8.9%	10.3%	-1.4%	18	11	49300	13.1%	-4.1%	9
Porirua City	49500	50600	51100	2.2%	3.2%	-1.0%	46	26	52400	5.9%	-3.6%	26
Upper Hutt City	37700	39700	37200	5.3%	-1.3%	6.6%	26	51	37900	0.5%	4.8%	52
Lower Hutt City	99100	101300	99300	2.2%	0.2%	2.0%	47	38	101300	2.2%	0.0%	41
Wellington City	171100	187700	180300	9.7%	5.4%	4.3%	16	19	183400	7.2%	2.5%	19
Masterton	23200	23200	23000	0.0%	-0.9%	0.9%	57	46	23400	0.9%	-0.9%	49
Carterton	7000	7260	7000	3.7%	0.0%	3.7%	30	39	7100	1.4%	2.3%	47
South Wairarapa	8900	9120	8900	2.5%	0.0%	2.5%	43	39	9100	2.2%	0.2%	40
Tasman	42400	45800	46300	8.0%	9.2%	-1.2%	19	13	47000	10.8%	-2.8%	14

Table 3 ctd.: **Comparison of SNZ 2001-base population projections published in November 2002 with actual 2006 population**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
			Med.		**Med.**			**Med.**	**High**	**High**		**High**
	EURP 30 June	**EURP 30 June**	**Proj. URP**	**Actual Pop Growth**	**Proj. Pop Growth**	**Actual-Proj**	**Actual Rank**	**Proj. Rank**	**Proj. URP**	**Proj. Pop Growth**	**Actual-Proj**	**Proj. Rank**
	2001	**2006**	**2006**	**2001-06**	**2001-06**	**2001-06**	**2001-06**	**2001-06**	**2006**	**2001-06**	**2001-06**	**2001-06**
Nelson City	42900	44300	45600	3.3%	6.3%	-3.0%	33	16	46300	7.9%	-4.7%	17
Marlborough	40700	43600	43100	7.1%	5.9%	1.2%	23	17	43800	7.6%	-0.5%	18
Kaikoura	3600	3730	3600	3.6%	0.0%	3.6%	31	39	3700	2.8%	0.8%	36
Buller	9900	9940	9600	0.4%	-3.0%	3.4%	54	61	9800	-1.0%	1.4%	61
Grey	13200	13550	12900	2.7%	-2.3%	4.9%	39	57	13200	0.0%	2.7%	54
Westland	8000	8620	7900	7.7%	-1.3%	9.0%	21	49	8100	1.3%	6.5%	48
Hurunui	10100	10750	10500	6.4%	4.0%	2.5%	24	23	10700	5.9%	0.5%	25
Waimakariri	37900	44100	43100	16.4%	13.7%	2.6%	5	5	44300	16.9%	-0.5%	4
Christchurch (incl. BP)	335200	361800	348100	7.9%	3.8%	4.1%	20	25	354100	5.6%	2.3%	27
Selwyn	28300	35000	31400	23.7%	11.0%	12.7%	2	10	32000	13.1%	10.6%	8
Ashburton	26000	28000	26100	7.7%	0.4%	7.3%	22	37	26600	2.3%	5.4%	39
Timaru	42800	43800	42200	2.3%	-1.4%	3.7%	44	52	42900	0.2%	2.1%	53
Mackenzie	3800	3900	3800	2.6%	0.0%	2.6%	40	39	3900	2.6%	0.0%	37
Waimate	7200	7380	6900	2.5%	-4.2%	6.7%	42	69	7000	-2.8%	5.3%	72
Chatham Islands	700	650	800	-7.1%	14.3%	-21.4%	73	4	800	14.3%	-21.4%	6
Waitaki	20500	20700	19700	1.0%	-3.9%	4.9%	52	65	20100	-2.0%	2.9%	68
Central Otago	14800	17050	14200	15.2%	-4.1%	19.3%	6	68	14600	-1.4%	16.6%	66
Queenstown-Lakes	17800	24100	22600	35.4%	27.0%	8.4%	1	1	23800	33.7%	1.7%	1
Dunedin City	119300	122300	122200	2.5%	2.4%	0.1%	41	32	124500	4.4%	-1.8%	31
Clutha	17600	17200	17100	-2.3%	-2.8%	0.6%	65	60	17500	-0.6%	-1.7%	59
Southland	29400	29200	28600	-0.7%	-2.7%	2.0%	59	58	29200	-0.7%	0.0%	60
Gore	12700	12400	12200	-2.4%	-3.9%	1.6%	66	66	12400	-2.4%	0.0%	71
Invercargill City	51100	51600	49700	1.0%	-2.7%	3.7%	51	59	51000	-0.2%	1.2%	58

Source: Statistics New Zealand data and Population Studies Centre calculations.

As with the previous periods described above, actual population growth was positive in most TLAs over the period 2001 to 2006, and was highest in Queenstown-Lakes District (35.4 percent) and Selwyn District (23.7 percent). Franklin District (14.3 percent) and Hamilton City (8.1 percent) also ranked within the nine fastest growing TLAs over that period, while South Waikato District (-4.1 percent) and Otorohanga District (-3.0 percent) both ranked in the bottom seven. In the SNZ 2001-base medium projection, the projected population growth was lower than actual population growth in 53 of the 73 TLAs, and with the exception of Central Otago District and Westland District, the difference between actual and projected population growth was greatest in the TLAs with the highest population growth. The population growth rate in Franklin District was higher than projected by 5.6 percentage points and Hamilton City by 4.4 percentage points.

Over this period there were some substantial differences between the projected and actual rankings of the TLAs in terms of population growth rates. The correlation coefficient between the actual and projected ranks is 0.710. This is lower than the corresponding correlation in Table 2 even though the projection is that table is over a longer period (9 years) than in Table 3 (4 years). Clearly, the relatively fast national population growth over the 2001-2006 period coincided with a considerable, and relatively more difficult to predict, dispersion of TLA growth rates. With the high projections (Columns (9) to (12) in Table 3), we find that projected population growth was lower than actual population growth in a smaller number of the 73 TLAs, 31 as compared with 53 in the medium projection. Among the Waikato TLAs, the high projection reduces the gap between the actual and the projected growth in the fast growing Franklin District and Hamilton City. Overall, the high projection does not improve the predicted ranking of TLA population growth: the correlation coefficient is 0.704, as compared with 0.710 for the medium projection.

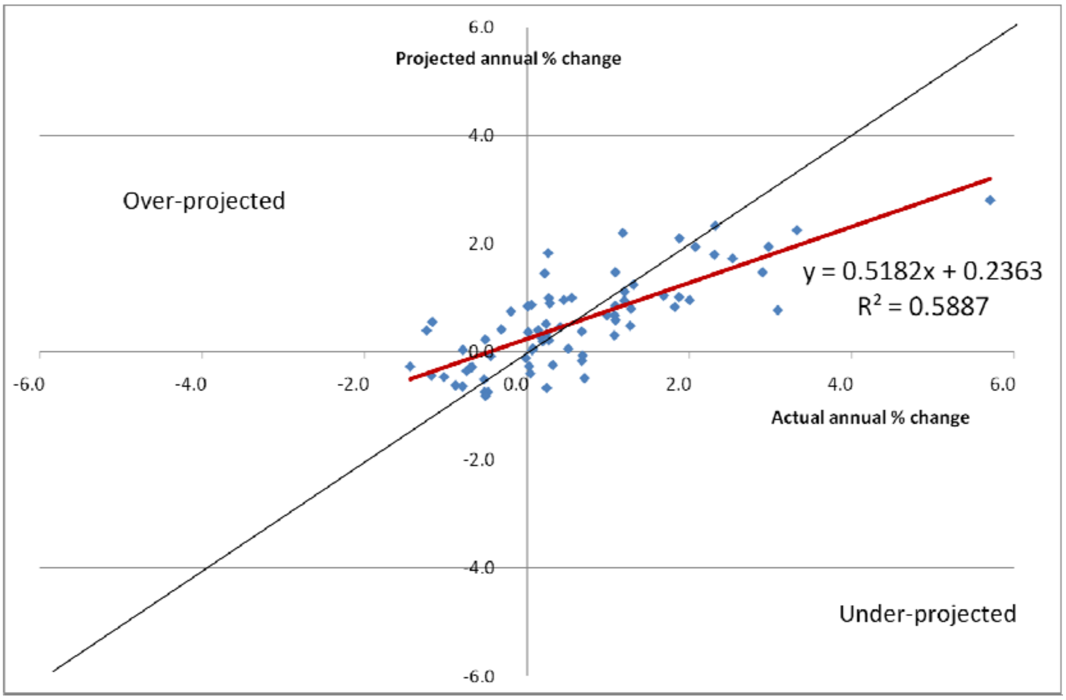
In general, the SNZ deterministic population projections are fairly good at predicting the *ranking* of sub-national areas in terms of population growth rates, but less good at predicting the *levels* of population growth. Figure 1 shows the 1991-base medium projected growth rates for territorial and local authorities in New Zealand over the period 1991 to 2006 against the actual population growth rates that obtained over that same period. The figure demonstrates that the deterministic projections are over-conservative as they have low Mincer-Zarnowitz efficiency (Mincer and Zarnowitz, 1969). Faster-growing areas are systematically under-projected, while slower-growing areas are systematically over-projected. This is consistent with SNZ's own recent analysis of the accuracy of population projections (Statistics New Zealand, 2008).

The inaccuracy of past population projections can only result from uncertainty in projections of natural increase (births minus deaths) or net migration. Figure 2 shows the contributions of net international migration and natural increase to population growth of New Zealand as a whole over the period 1991 to 2007. Three features are notable from this figure. First, the increase in population is mostly determined by natural increase, which on average contributes about two thirds of population growth, even over this period with historically high net international migration.[6] Natural increase is also relatively stable at about 0.7 percent over this period. Second, the

[6] For all the years since 1950, net international migration in aggregate accounted for only one quarter of population growth in New Zealand and natural increase for three quarters (see Poot, 2008).

major source of the observed volatility in population growth is net international migration, which has varied between 1.0 percent (a net in-migration of 38,198 in 2002) and -0.3 percent (a net out-migration of 11,312 in 2000) over this period. Net migration is obviously cyclical, but both the duration and the amplitude of the cycles are rather difficult to predict. A major contributor is Trans-Tasman migration (e.g. Gorbey *et al.*, 1999) and there appears to be a decadal cycle in which the net trans-Tasman outflow is greater in the second half of recent decades than in the first half (Poot, 2010). Third, the greatest absolute levels of net international migration have occurred in the five year period up to 2006, and in 2002 and 2003 in particular. Up until that time the contribution of net international migration to population growth had been lower than that of natural increase, at least since 1991.

Figure 1: Comparison of projected vs. actual population growth rates for territorial and local authorities, SNZ 1991-base sub-national population projections

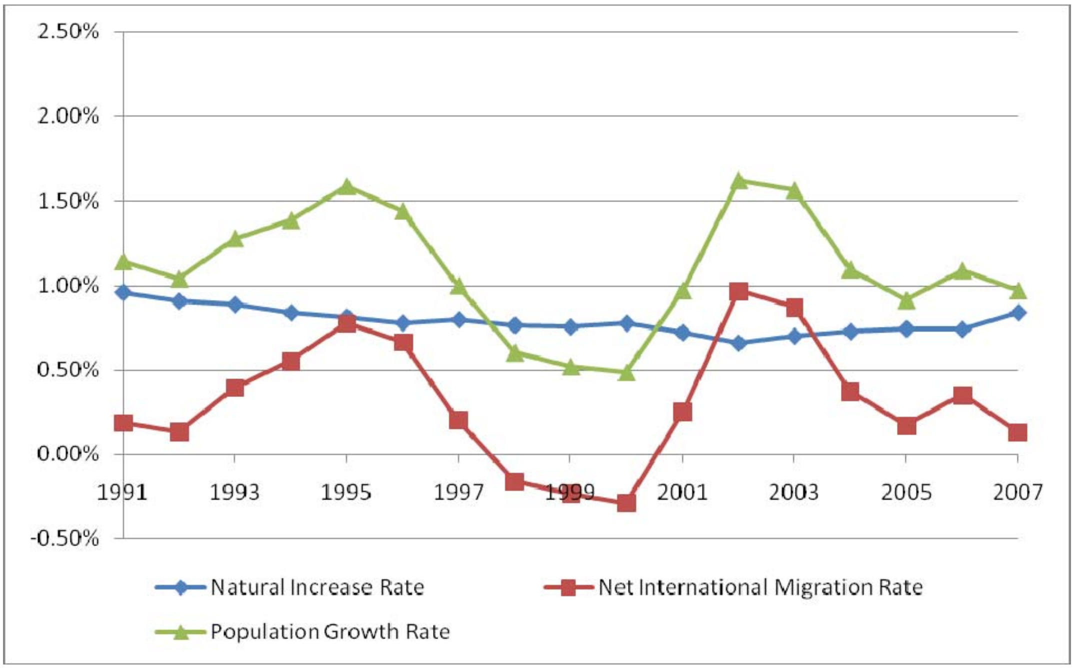


Source: Statistics New Zealand data and Population Studies Centre calculations.

These features provide the key explanation for the observed differences between actual and projected population growth rates noted earlier. The relatively stable rate of natural increase confirms that fertility and mortality assumptions derived from past trends are unlikely to contribute significantly to the observed differences between actual and projected population growth over a relatively short to medium time horizon, such as between 5 to 15 years. Fertility and mortality assumptions can of course have a much bigger impact over longer time horizons, but the volatility of net international migration presents an obvious challenge to the development of plausible migration assumptions for population projections over a short to medium time frame.

Further, in the period between 2001 and 2004 New Zealand experienced an exceptional increase in net international migration, which could not have been predicted by solely considering past trends. In contrast, on average net international migration was lower between 1991 and 2000 and provided only a small contribution to population growth. This alone is likely to have resulted in much of the observed differences between actual and projected population growth.

Figure 2: New Zealand population growth, 1991-2007 (December Years)



Source: Statistics New Zealand data.

Table 4 shows the impact of net international migration on each region in New Zealand over the period 2003 to 2008, and provides further evidence of the volatility of net migration flows. Over the period 2003 to 2008, the Waikato region experienced annual net international migration that varied between 1,402 in 2003 and -1,116 in 2008.[7] This high level of variability is illustrated by the standard deviation of these net in-migration flows being more than seven times the mean net migration flow of -95 over that period.

The experience of New Zealand since 2001 presents a particular problem for the preparation of population projections – is that experience indicative of a permanent change in the long-run level of net international migration, or is it simply an unusually high cyclical peak, with the net international migration rate continuing to fluctuate around the long-run average of 0.3 percent per year? Table 4 provides some guidance on this. Since 2004, net international migration has decreased from the peaks of 2002 and 2003, with net international migration falling to just 3,814 for the 2008 calendar year, the lowest since 2000. However, it would be unwise to assume that this

[7] Of course, net internal migration makes also a significant contribution to population growth.

represented a return to a ‘normal’ lower level of net international migration. During 2009 net in-migration surged up to 21,253, the highest annual net in-migration since 2003. This does not mean that immigration increased during 2009. Consistent with the decline in international migration around the world resulting from the global economic downturn (e.g., Fix *et al.*, 2009) immigration to New Zealand and emigration from New Zealand both declined, but the decline in Permanent and Long-Term (PLT) departures (from 83,649 to 65,157) far exceeded the decline in PLT arrivals (from 87,463 to 86,410). The time series dynamics of emigration and immigration are often quite different and the recent experience illustrates that it will be important in the future to consider emigration and immigration separately in the projection methodology. Wilson (2005) made a first start with this approach in his stochastic national projections for New Zealand. However, he modelled immigration separately, but then calculated emigration by means of simply subtracting net migration, which was modelled on its own as well. In the present paper, we adopt for convenience the conventional approach of focussing on net migration, but noted already in Section 2 that this may be theoretically and empirically less attractive than models of gross migration (Rogers, 1990). Future development of the methodology will include separate stochastic modelling of immigration and emigration.

Table 4: Net international migration by New Zealand region of residence

	2003	**2004**	**2005**	**2006**	**2007**	**2008**
Northland	174	-91	-268	-277	-452	-1,026
Auckland	17,801	9,499	6,352	9,806	6,405	5,871
Waikato	1,402	234	-436	-84	-571	-1,116
Bay of Plenty	237	-420	-1,146	-772	-1,231	-1,883
Gisborne	90	-114	-132	-123	-223	-464
Hawkes Bay	51	-197	-383	-204	-490	-1,097
Taranaki	253	-73	-62	-5	-93	-165
Manawatu-Wanganui	547	133	-181	100	-661	-624
Wellington	1,657	620	-37	1,035	-508	-197
Tasman	98	65	-132	-143	-123	-242
Nelson	258	37	180	118	52	49
Marlborough	85	49	-34	-33	128	-66
West Coast	48	-22	-23	52	-76	57
Canterbury	4,108	2,221	1,295	1,993	745	948
Otago	1,018	673	425	278	202	-128
Southland	212	-38	-131	-81	-178	-51
Other/Not Stated	6,867	2,532	1,684	2,949	2,565	3,948
TOTAL	**34,906**	**15,108**	**6,971**	**14,609**	**5,491**	**3,814**

Source: Statistics New Zealand data

4 Demographic Change in the Waikato Region

As noted earlier, we present in this paper an application of the stochastic cohort-component method to several small sub-national populations in the Waikato region of New Zealand. The Waikato region is composed of all or part of 14 TLAs, with an estimated resident population of 383,716, or 9.5 percent of the New Zealand population at the time of the 2006 Census. It is a region of significant demographic diversity (see Table 5), including New Zealand's fourth largest city by population (Hamilton), peri-urban districts with close urban ties (such as Franklin District on the edge of Auckland, New Zealand's largest city), a coastal district with a small permanent population but large transient population flows (Thames-Coromandel District), and several small peripheral and predominantly rural districts (such as Otorohanga District and South Waikato District). Some TLAs have recently experienced significant population growth, while others are experiencing a general decline in population. International migration is particularly affecting the metropolitan area of Hamilton city and its surrounding areas, but far less so rural and peripheral areas such as South Waikato and Waitomo (see Table 5). This diversity makes the Waikato region an ideal subject area for a study into the results and implications of sub-national stochastic population projections under a variety of initial and future conditions.

Current sub-national population projections for New Zealand are subject to considerable uncertainty, particularly in terms of future migration (Cameron *et al.*, 2008a). This is generally in line with the experience of other similar countries in population projections generally (Shaw, 2007; Wilson, 2007). The uncertainty of net migration is a key feature of sub-national population projections in New Zealand, which are subject to both uncertain and highly volatile international migration flows and uncertain internal migration flows. The huge change in net migration between 2008 and 2009 discussed in the previous section is a recent example.

5 The Stochastic Cohort-Component Methodology

Deterministic projections are often adopted in preference to stochastic projections due to their simplicity, transparency, and low cost (e.g. the projections can be run in a simple spreadsheet). However, as noted earlier, deterministic projections may be too conservative due to the subjective choice of assumptions, often linked to recent trends, in what is an inherently scenario-based method. For instance, SNZ develops sets of national projections based on low, medium, and high projected series for each of fertility, mortality, and net migration. Deterministic scenarios often implicitly assume some correlation between the parameter values that were selected for the projected series, and for pragmatic reasons only a subset of possible scenarios based on the low, medium, and high series are ever developed. Under this method, a 'high projection' scenario might assume high fertility, low mortality, and high net migration, while a 'low projection' scenario might assume low fertility, high mortality, and low net migration. For example, the combination of low fertility, low mortality and high net migration might never be considered.

Table 5: Total population (5 years and over), inward international migration and internal migration for Waikato territorial authority areas and Waikato Region, 2001-2006

Area	2006 population aged 5+ (000s)	In-migration from within NZ (%)	In-migration from overseas (%)	Stayed within TLA (%)	Not specified (%)
Franklin	58.9	27.7	6.0	63.4	2.9
Thames-Coromandel	25.9	29.4	5.2	62.3	3.0
Hauraki	17.2	31.2	3.7	61.2	4.0
Waikato	44.0	34.1	4.5	57.8	3.6
Matamata-Piako	30.5	24.2	4.4	68.1	3.4
Hamilton	129.2	25.7	10.3	61.6	2.4
Waipa	42.5	27.3	6.5	63.3	2.8
Otorohanga	9.0	32.3	3.5	58.9	5.3
South Waikato	22.6	24.6	3.5	68.2	3.7
Waitomo	9.4	26.8	3.3	65.7	4.2
Taupo	32.4	27.7	5.3	63.6	3.4
Rotorua	65.9	21.8	6.1	68.5	3.7
WAIKATO REGION	**382.7**	**13.5**	**6.7**	**76.8**	**3.1**

Source: Statistics New Zealand data and Population Studies Centre calculations.

However, a stochastic methodology allows many more combinations of fertility, mortality, and net migration to be explored, along with explicitly taking account of the probabilities of each ‘joint scenario’ occurring.[8]

Furthermore, the increasing variability of migration flows suggests that alternative methods for population projections need to be used in order to capture this inherent uncertainty. Stochastic projection techniques can explicitly account for both spatial and temporal variation in uncertainty by incorporating quantitative differences in the underlying parameter distributions as well as explicitly modelling both trends and cycles in the parameters. Stochastic projections also have advantages in the types of policy questions that can be answered – for instance, probabilistic statements can be

[8] A review of some stochastic simulation techniques and applications can be found in a special issue of the *New Zealand Population Review*, see Dharmalingam and Pool (2005).

made about the probability that the population will be between *x* and *y*. Probabilistic statements about derived indicators such as the number of new entrants at schools or the age dependency ratio can also be made.

Wilson (2005) provides an example of national probabilistic projections in New Zealand. Stochastic population projection techniques are far less commonly applied to the case of regional projections than to national projections. In the few cases where the regional level is considered, the application tends to be for a single region (see for example the case study of Queensland, Australia, by Wilson and Bell (2007)). Here we present an application of the stochastic cohort-component method to several territorial and local authorities in the Waikato region. This represents an extension of the simple cohort-component model applied in Cameron *et al.* (2008b) for Thames-Coromandel District.

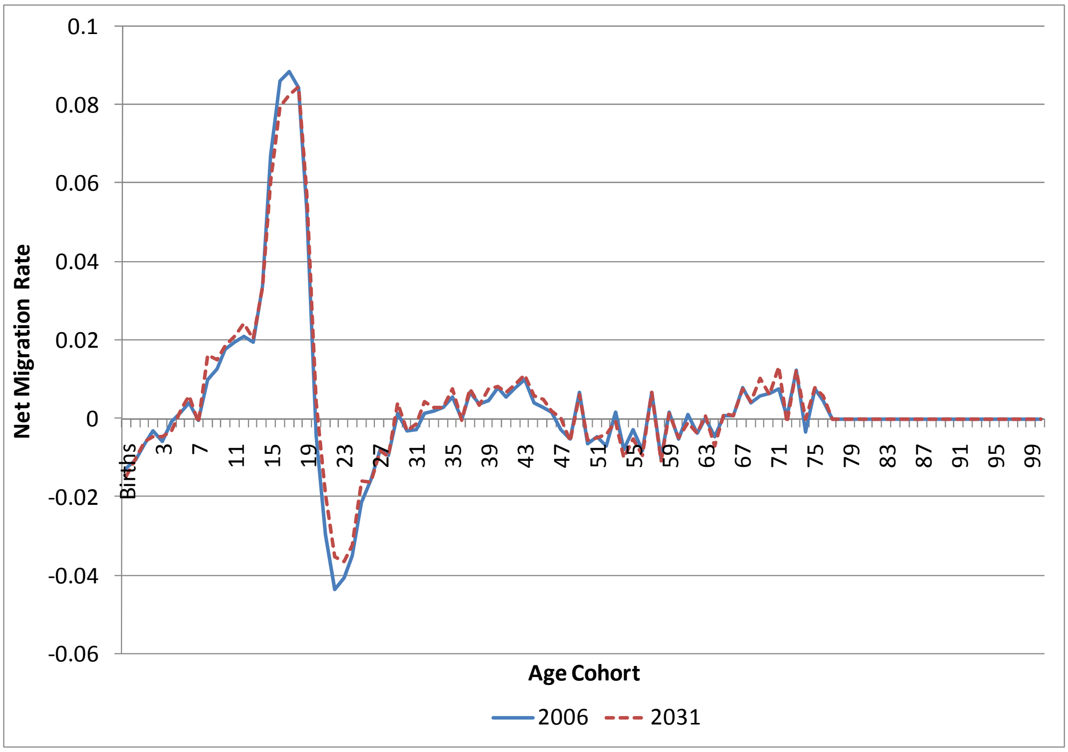
In Cameron *et al.* (2008b), the 'medium' age-specific fertility and survivorship (mortality) assumptions of Statistics New Zealand (SNZ) were used, but an alternative approach was adopted for the estimation of net migration. For sub-national projections, the projection methodology employed by SNZ involves assuming a total value for net migration for each territorial authority in each year, which is allocated across ages and gender. The alternative approach involves assuming gender- and age-specific net migration *rates*. Under this method, the projected level of net migration reflects a combination of the projected net migration rates which vary over time and the population to which these rates are applied, which also varies over time in a similar way to how fertility and survivorship rates affect births and deaths respectively. Two alternative methods for estimating historical gender- and age-specific net migration rates, involving net residual and regression methods, are described in Cameron (2010). For the purposes of this paper, the net migration rates were estimated using the regression method, which has better in-sample properties (Cameron, 2010).

This paper extends the method employed in Cameron *et al.* (2008b), which was a deterministic projections method, by applying assumptions about the *distributions* of projected fertility, survivorship, and net migration rates. This allows stochastic projections to be computed, by repeatedly drawing at random different combinations of fertility, survivorship, and net migration rates from their respective distributions. The distributions were generated using a combination of time series models and expert judgement, with data for developing the distributions obtained from Statistics New Zealand. The equations of the stochastic projection methodology are given in the Appendix.

The distributions and draws were undertaken as follows. First, a deterministic forecast of each age- and gender-specific rate (fertility, survivorship, and net migration) was developed. Deterministic age- and gender-specific fertility and survivorship rates were derived from the medium series assumptions used in the SNZ 2006-base sub-national population projections. Net inter-censal migration rates were estimated using the regression method as detailed in Cameron (2010), and projected using a three-period moving average. They were then converted to annual rates, smoothed across five-year age groups, and calibrated so that projections, which have 2006 as the base year, would closely replicate the 2008 sub-national population estimates prepared by

SNZ in total population and population by gender.[9] An example of the resulting age-specific net migration profiles, for males in Hamilton City in 2006 and 2031, is shown in Figure 3. The profile is consistent with *a priori* expectations about net migration into Hamilton City, with significant in-migration during school and university-entrance ages, and significant out-migration following the typical age of completion of undergraduate study. There is also net inward migration of people around age 40 and of those in their early 70s, likely to be associated with retirement. The age profile for females is similar. These net migration profiles are basically stable over time.

Figure 3: Age-specific net migration profile for Hamilton City males, 2006 and 2031



Second, each age- and gender-specific rate was multiplied by a factor which is probabilistic. The percentage change in each of the rates is given by *k*, whereby *k* is drawn independently from a separate distribution for fertility, mortality and migration. The entire deterministic path of fertility, mortality and net migration rates over the 2006-2031 projection period was shifted by the corresponding factors.[10] In this way, if all multipliers were set to 0 this would result in the deterministic projection and the multiplier is varied around 0 to increase or decrease each rate. For net migration rates, the effect of the multiplier further depends on the sign of the rate. Greater net

[9] While it may be preferable to calibrate to the sub-national population estimates by age cohort, the significant rounding of the reported estimates could lead to significant over- or under-calibration of the resulting net migration rates over such a small time period (two years). Given this possibility, calibration was only performed to the total and gender-specific estimated populations.

[10] In principle it is also possible to introduce stochasticity by drawing random paths, such as by modelling each time series of rates as a mean-reverting (stationary) process, and drawing random 'shocks' to the path from a distribution, with the effect of each individual shock on the series reducing over time.

migration is assumed to increase in-migration among gender- and age-specific groups that migrate into an area, and to decrease out-migration among gender- and age-specific groups that migrate out of an area. Further details are given in the Appendix.[11]

Third, a distribution of *k* was determined for each of fertility, survivorship, and net migration, based on patterns of historical data. At the national level, data on component rates of population change over the period 1950 to 2009 are presented in Table 6. Taking 3 standard deviations from mean as the upper and lower bounds, the crude birth rate varies within 5 to 9 percent of its long-run average, the crude death rate varies within 3 to 4 percent of its long-run average, and the crude net migration rate varies within 67 to 72 percent of its long-run average.

Distributional assumptions for each multiplier were based on these data, as follows. The fertility multiplier was assumed normally distributed with a mean zero and standard deviation of 1.67 (giving a range of about +/- 5% of the mean fertility rates). The survivorship multiplier was assumed normally distributed with mean zero and a standard deviation of 0.67 (i.e., giving a range of +/- 2% of the mean mortality rates) The net migration multiplier was assumed normally distributed with mean zero and a standard deviation of 16.67 (i.e., giving a range of +/- 50% of the mean net migration rates. In all cases, the assumed variability is similar or somewhat less than that observed over the periods since 1950 and since 1991.

Table 6: Component rates of population change in New Zealand, 1950-2009

	1950-2009			1991-2009		
	Mean	**Mean ± 3 SD**	**3SD/Mean**	**Mean**	**Mean ± 3 SD**	**3SD/Mean**
Crude birth rate (%)	1.942	(1.773,2.112)	8.7%	1.545	(1.469,1.622)	5.0%
Crude death rate (%)	0.819	(0.791,0.848)	3.5%	0.732	(0.705,0.759)	3.7%
Crude net migration rate (%)	0.304	(0.102,0.507)	50.7%	0.495	(0.140,0.851)	71.8%

Source: Statistics New Zealand data and Population Studies Centre calculations.

Finally, a random draw was taken independently from each distribution of the multipliers (fertility, survivorship, and net migration) for each run of the projections. While in principle any covariance between fertility, mortality, and net migration can be taken into account when making a draw from each distribution, this was not done in this case. For each territorial and local authority, the same random draw (same random seed) was applied in each run of the projections.[12] In total, the projections

[11] This method of introducing stochasticity into the projection model has the advantage that policy-makers can easily specify new deterministic scenarios with an *x*% change in fertility rates, a *y*% change in mortality rates, and a *z*% change in net migration rates from the base deterministic scenario. This is important in applications of the model, such as its use in the Waikato Integrated Scenarios Explorer model (Huser *et al.*, 2009).

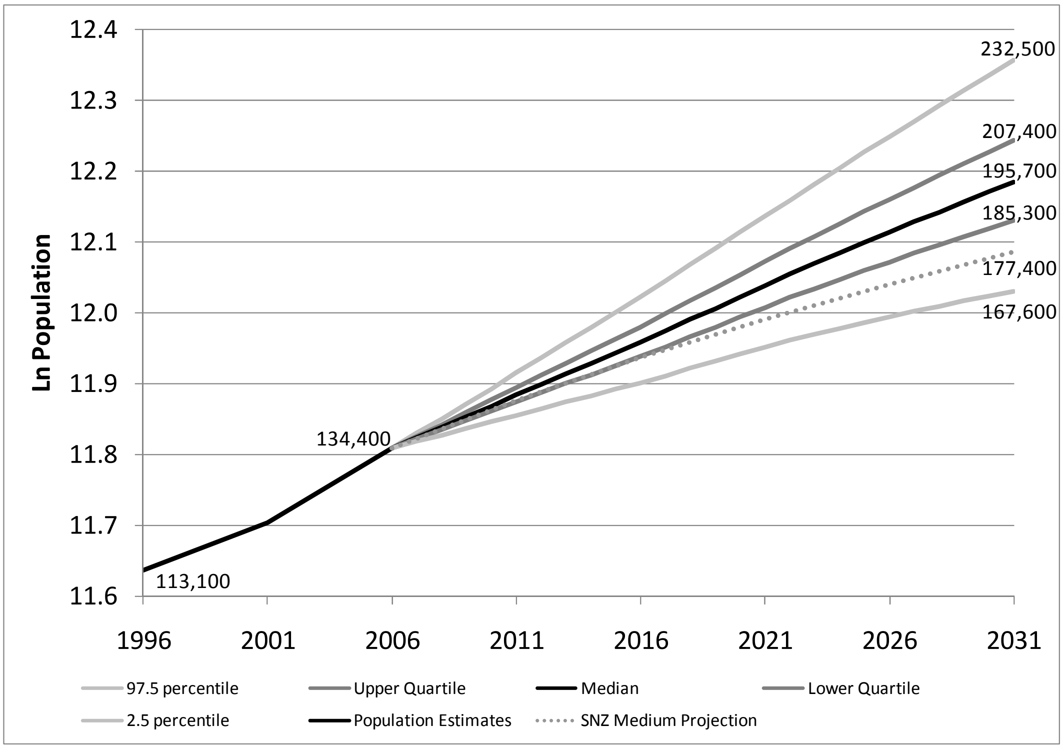
[12] In other words, a perfect correlation in the random draws within each distribution between each TLA was assumed.

were run 10,000 times by repeatedly drawing at random different combinations of fertility, mortality, and net migration multipliers. The resulting stochastic population projections for each sub-national area considered are presented in the next section.

6 Sub-National Stochastic Population Projections 2006(base)-2031

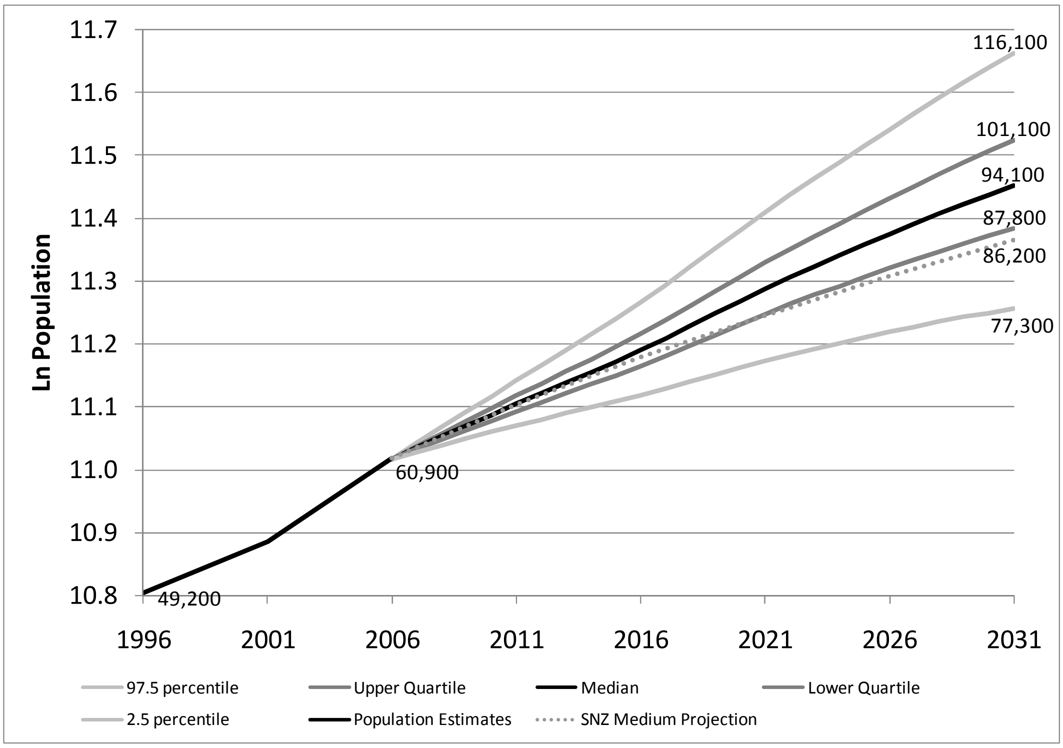
The stochastic projection of population for Hamilton City to 2031 is presented in Figure 4, with the deterministic SNZ 2006-base medium population projection for comparison. A natural logarithmic scale is used to easily gauge the expected population growth. Hamilton City is the main urban centre in the Waikato region, with a 2006 estimated usually resident population of 134,300. The stochastic projection has a median population of 195,700 in 2031, with an inter-quartile range of 185,300 to 207,400 and a 95 percent confidence interval of 167,600 to 232,500. The median projected annual population growth rate is 1.5 percent per annum. From the figure the conservatism of the medium deterministic projection is clear – it lies outside the inter-quartile range of the stochastic projection (but within the 95 percent confidence interval).

Figure 4: Stochastic population projection for Hamilton City, 2006-2031



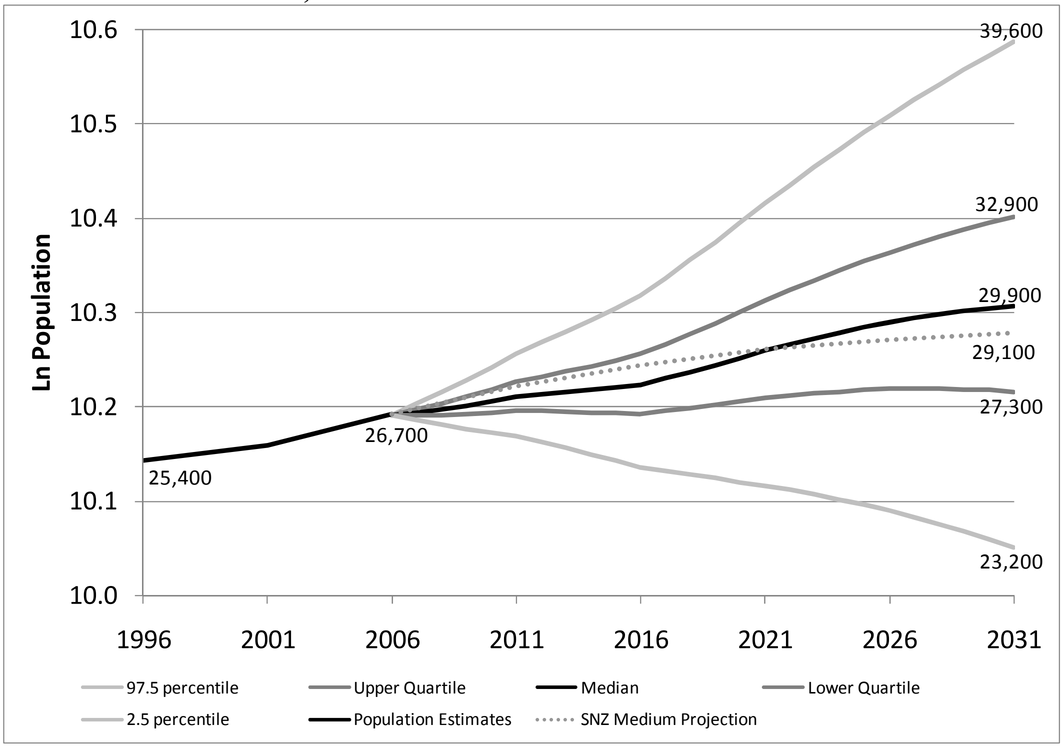
The stochastic projection of population for Franklin District to 2031 is presented in Figure 5, with the deterministic SNZ 2006-base medium population projection for comparison. Franklin District is a peri-urban district that borders on Auckland, New Zealand's largest city, and had a 2006 estimated usually resident population of 60,900. The stochastic projection has a median population of 94,100 in 2031, with an inter-quartile range of 87,800 to 101,100 and a 95 percent confidence interval of 77,300 to 116,100. The median annual projected population growth rate is 1.8 percent per annum. Again the medium deterministic projection is clearly conservative, lying just outside the 50 percent confidence interval of the stochastic projection.

Figure 5: Stochastic population projection for Franklin District, 2006-2031



The stochastic projection of population for Thames-Coromandel District to 2031 is presented in Figure 6. Thames-Coromandel District is a coastal area with many small communities and a permanent population that is small relative to the peak population in holiday periods (Cameron *et al.*, 2008b), and had a 2006 estimated usually resident population of 26,700. The stochastic projection has a median population of 29,900 in 2031, with an inter-quartile range of 27,300 to 32,900 and a 95 percent confidence interval of 23,200 to 39,600. The median projected annual population growth rate is 0.5 percent per annum, and there is a projected 19.5 percent chance of population decline in the district between 2006 and 2031. The deterministic projection is much closer to the median stochastic projection for this district, being 29,100 in 2031.

Figure 6: Stochastic population projection for Thames-Coromandel District, 2006-2031



The stochastic projection of population for Otorohanga District to 2031 is presented in Figure 7. Otorohanga District is a predominantly rural area with no large population centre, with a 2006 estimated usually resident population of 9,310. The stochastic projection has a median population of 8,410 in 2031, with an inter-quartile range of 7,780 to 9,140 and a 95 percent confidence interval of 6,740 to 10,770. The median projected annual population growth rate is -0.4 percent per annum, and there is a projected 19.8 percent chance of a population increase in the district between 2006 and 2031. The deterministic projection (8,350) is very close to the median stochastic projection.

The stochastic projection of population for South Waikato District to 2031 is presented in Figure 8, with the deterministic SNZ 2006-base medium population projection for comparison. South Waikato District is a rural area centred on a single large population centre (Tokoroa), with a 2006 estimated usually resident population of 23,200. The stochastic projection has a median population of 18,500 in 2031, with an inter-quartile range of 17,400 to 19,700 and a 95 percent confidence interval of 15,700 to 22,300. The median projected annual population growth rate is -1.0 percent per annum, and there is just a projected 0.9 percent chance of a population increase in the district between 2006 and 2031. Again the medium deterministic projection (18,450) is very close to the median stochastic projection by 2031, but is much higher over most of the projection period and outside the 50 percent confidence interval until about 2016.

Figure 7: Stochastic population projection for Otorohanga District, 2006-2031

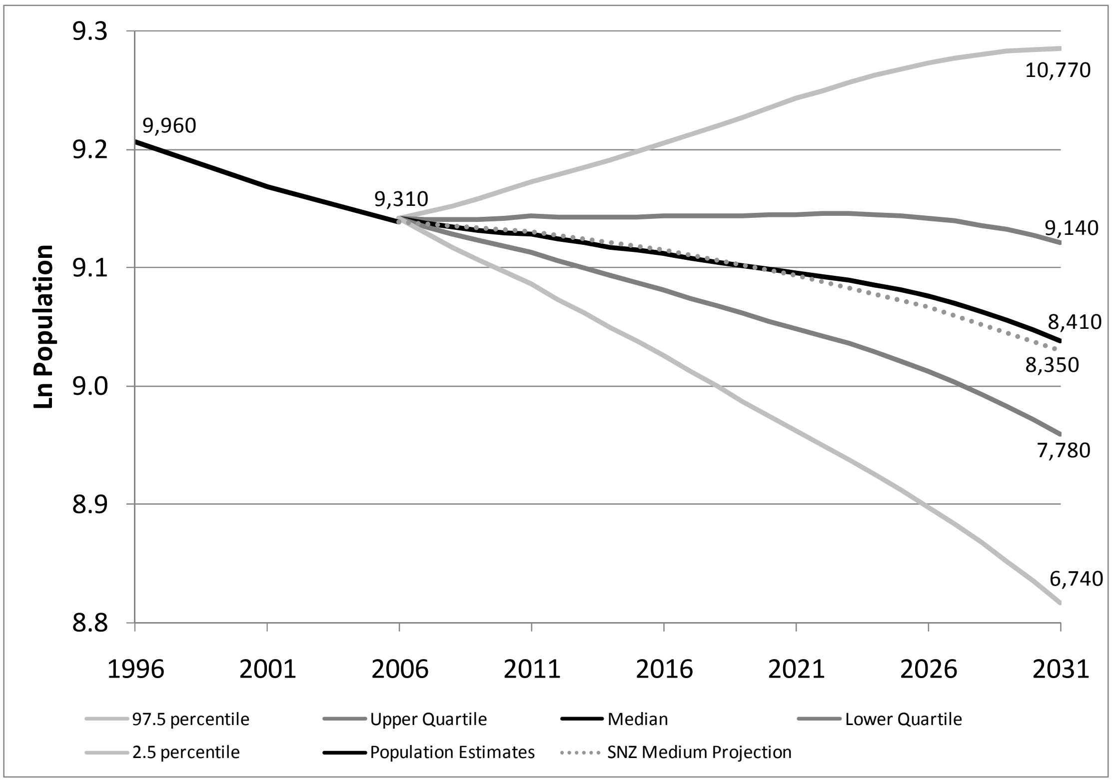
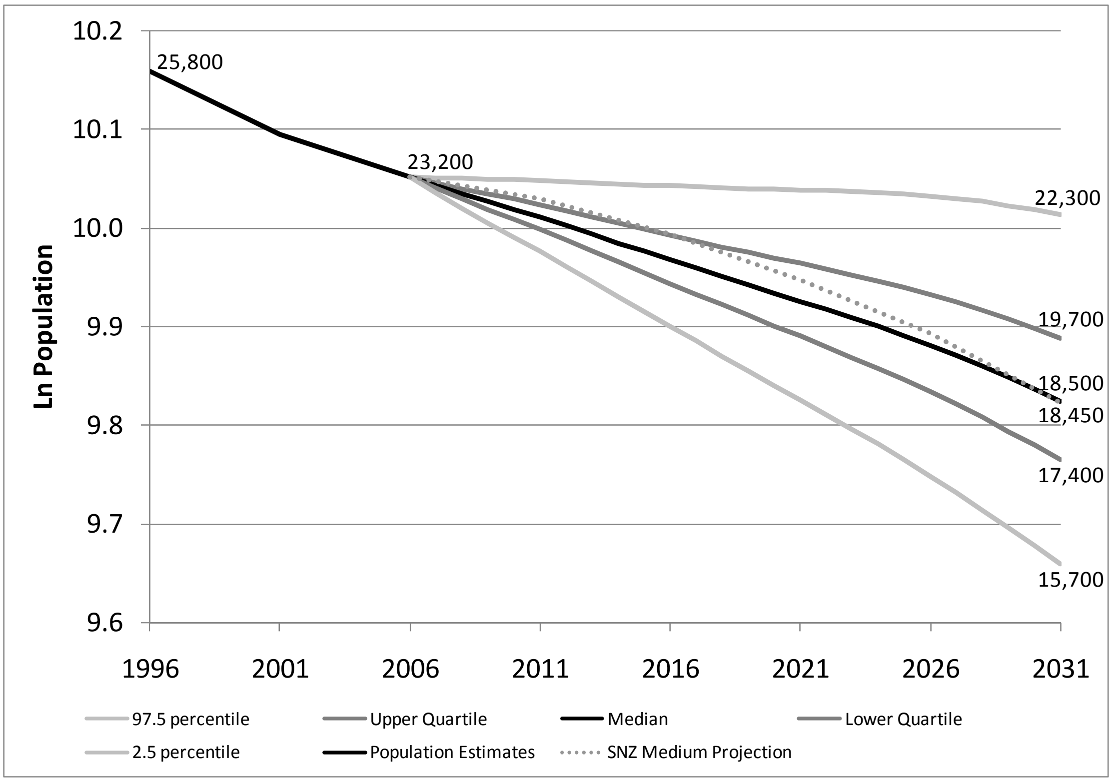


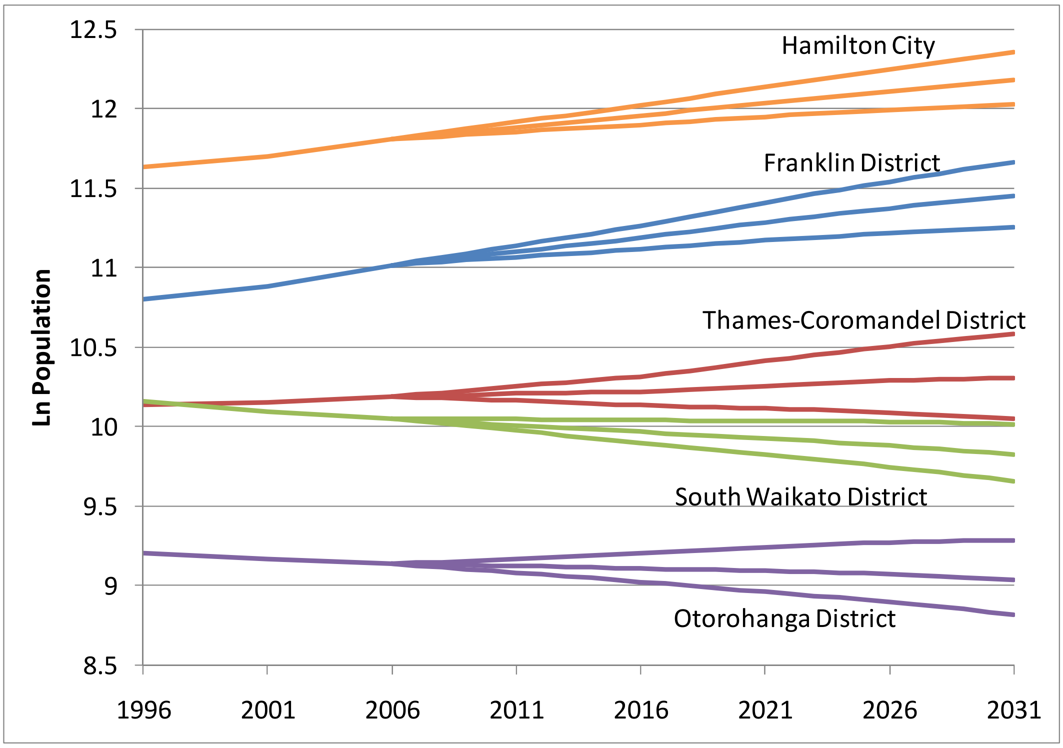
Figure 8: Stochastic population projection for South Waikato District, 2006-2031



From the above results, it is clear that the SNZ medium deterministic projection is most conservative for the districts that are projected by the stochastic model to grow the fastest, and more consistent with the median stochastic projection for those districts projected to grow the slowest or decline. This is consistent with previous projections, where the fastest growing districts were systematically under-projected (see Section 3 above), although it is less clear that the slow growing and declining regions are over-projected. Nonetheless, the results suggest that conservatism with respect to faster-growing sub-national areas continues to present a very real problem for deterministic projections.

Also, the 'spread' or range of the stochastic projections is largest for the districts that are the smallest and have the most stable populations in relative terms, i.e. those that are growing or declining the slowest. Figure 9 combines the median and 95 percent confidence interval projections for each of the five territorial authorities on the same scale, and demonstrates that the widest uncertainty bounds occur for the districts that are smallest and have the most stable median projected populations.

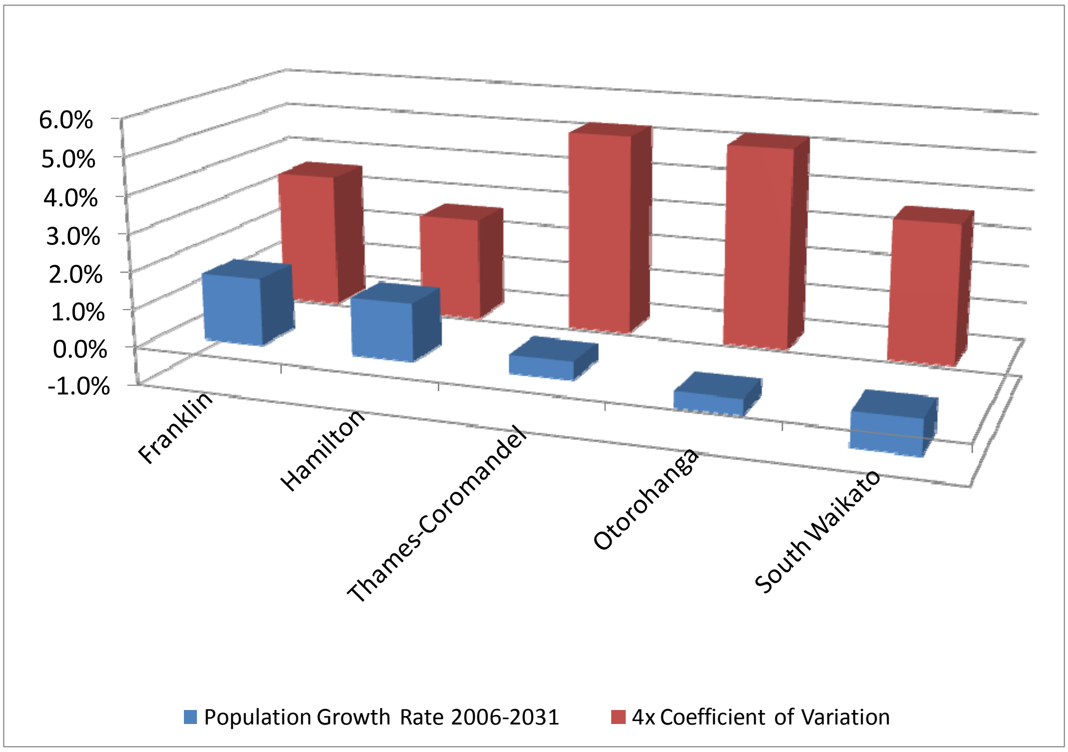
Figure 9: A comparison of stochastic population projections for selected Waikato Territorial and Local Authority regions, 2006-2031



The uncertainty in projections is further demonstrated by Figure 10, which presents the projected median annual population growth rate of the stochastic projections over the period 2006-2031 and four times the coefficient of variation of the stochastic projections (i.e. approximately the width of the 95 percent confidence interval as a proportion of the median projection). The figure clearly demonstrates that relatively high uncertainty (as measured by four times the coefficient of variation) occurs for those districts that have relatively small populations and little population change in the median projection, i.e. Thames-Coromandel District, Otorohanga District and

South Waikato. The projections suggest that Hamilton City will have relatively fast growth, but with relatively little uncertainty.

Figure 10: Median population growth rates and projection uncertainty, 2006-2031



7 Conclusions

Past deterministic sub-national population projections produced by SNZ have proven to be over-conservative, systematically under-projecting the populations of the fastest-growing areas and over-projecting the populations of the slowest-growing and declining areas. The source of this bias is the use of conservative estimates of net migration flows, which present the largest component of short-term volatility in population change.

Sub-national stochastic population projections, even using relatively narrow distributions of future fertility, mortality, and net migration rates, illustrate the considerable uncertainty associated with population projections. In relative terms, the uncertainty is greatest for the smallest districts, and those with low or declining population growth rates. Under these conditions of considerable uncertainty, the traditional ‘medium’ scenario of sub-national population projections will be of limited use for policy analysis or planning beyond a relatively short projection horizon.

8 Future developments of stochastic population projection methods at the Population Studies Centre

The stochastic population projections method employed in this paper is the first stage of an ongoing development of these methods at the Population Studies Centre at the University of Waikato. Several future developments are planned, including: (1) the drawing of samples of paths of time-varying parameters rather than random sample of parameters that are maintained throughout the projection period; (2) separating out international and internal inward and outward migration, combined with the generation of stochastic paths of immigration and emigration rates, with bounds on net migration imposed (or modelled covariance between the rates), and generation of internal migration rates by means of a gravity model; and (3) development of more realistic stochastic projection models that account for the interrelationship between fertility, mortality and migration parameters, rather than drawing from each distribution independently.

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Appendix: Stochastic Population Projection Model Equations

The stochastic population projection model uses just three sets of equations in order to generate population projections.[13]. The general formula for deriving the population at each year-of-age and each gender at each location is as follows:

$$P_{agl}^{t+1} = \begin{cases} a=0: P_{0gl}^{t+1} = \frac{1}{2}(B_{gl}^{t} + N_{-1gl}^{t}) \times S_{-1gl}^{t} + \frac{1}{2}(B_{gl}^{t} + N_{-1gl}^{t}) \\ 1 \le a \le 99: P_{agl}^{t+1} = (P_{(a-1)gl}^{t} + \frac{1}{2}N_{(a-1)gl}^{t}) \times S_{(a-1)gl}^{t} + \frac{1}{2}N_{(a-1)gl}^{t} \\ a=100: P_{100gl}^{t+1} = (P_{99gl}^{t} + \frac{1}{2}N_{99gl}^{t}) \times S_{99gl}^{t} + \frac{1}{2}N_{99gl}^{t} + (P_{100gl}^{t} + \frac{1}{2}N_{100gl}^{t}) \times S_{100gl}^{t} + \frac{1}{2}N_{100gl}^{t} \end{cases}$$

Where: P_{agl}^{t+1} is the population of age *a* and gender *g* in location (TLA) *l* at time *t*+1 (with *t* measured in years);

P_{a-1gl}^{t} is the population of age *a*-1 and gender *g* in location (TLA) *l* at time *t*;

B_{gl}^{t} is the number of births of gender *g* in location (TLA) *l* between time *t* and time *t*+1;

$N_{(a-1)gl}^{t}$ is the net migration of people of age *a*-1 and gender *g* to/from location (TLA) *l* between time *t* and time *t*+1; [14]

$S_{(a-1)gl}^{t}$ is the survivorship rate for people who are of gender *g* and in location (TLA) *l* and age *a*-1 at time *t*, who survive to age *a* at time *t*+1; [15]

subscripts *a* have a range of -1 to 100, with -1 representing births during the previous twelve months, 0 representing those of age 0, 1 representing those of age 1, …, and 100 representing those aged 100 or over;

subscripts *g* refer to gender, with 1 representing male and 2 representing female; and

subscripts *l* are integers 1, 2, ... with each number representing one of the selected TLAs in the Waikato region.[16]

The main additional assumption implicit in this formula is that migration, births, and deaths are all evenly spaced throughout the year. This allows half of the migrants and half of the births to be subject to the full year's survivorship rate. It should also be emphasised that the age group represented by *a*=100 is actually all people of gender *g* in location *l* aged 100 or over.

Births are calculated using the following formula:

[13] Actually, the population model as currently programmed in Vensim (Ventana Systems, 2005) uses many more equations than this, but in the model each of the equations presented here is broken down into constituent parts for ease of calculation.

[14] A positive number for net migration represents net in-migration, while a negative number for net migration represents net out-migration.

[15] That is, the fraction of people of gender *g* and in location *l* who were age *a*-1 at time *t* and who are still alive (and therefore aged *a*) at time *t*+1.

[16] The Waikato region consists of 12 TLAs: 1=Franklin, 2=Thames-Coromandel, 3=Hauraki, 4=Waikato, 5=Matamata-Piako, 6=Hamilton City, 7=Waipa, 8=Otorohanga, 9=Waitomo, 10=South Waikato, 11=Taupo, 12=Rotorua.

$$B_{gl}^t = \begin{cases} g = 1 : B_{1l}^t = G_l^t \times \sum_{a=13}^{49} (F_{al}^t \times (P_{a2l}^t + \frac{1}{2} N_{a2l}^t)) \\ g = 2 : B_{2l}^t = (1 - G_l^t) \times \sum_{a=13}^{49} (F_{al}^t \times (P_{a2l}^t + \frac{1}{2} N_{a2l}^t)) \end{cases}$$

Where: G_l^t is the fraction of births between time t and time t+1 that are male (the masculinity ratio of births) in location (TLA) l; and

F_{al}^t is the fertility rate for women of age a in location (TLA) l between time t and time t+1.

It should be noted that in the current model the sex ratio of births is assumed to be constant both across time and between TLAs at 105.5 male children for every 100 female children, i.e. a masculinity ratio of 0.5134 This is consistent with the experience of New Zealand over the past several decades. Second, only the fertility of women between the ages of 13 and 49 are considered as women outside that age group have very few children.

Net migration for each year-of-age and each gender is calculated using the following formula:

$$N_{agl}^t = \begin{cases} a = -1 : (M_{-1gl}^t \times B_{gl}^t) \\ a \geq 0 : (M_{agl}^t \times P_{agl}^t) \end{cases}$$

Where: M_{agl}^t is the migration rate of people of age a and gender g to/from location (TLA) l between time t-1 and time t.

Multipliers

The survivorship rates, fertility rates, and migration rates in the population model can all be varied randomly within a distribution in order to generate stochastic projections. The stochastic parameters, i.e. the parameters that are drawn randomly from certain distributions for each projection "run", are introduced in the model using the following additional formulae:

$$F_{al}^t = f_{al}^t \times (1 + \frac{k_{al}^f}{100})$$

$$S_{agl}^t = 1 - (1 - s_{agl}^t) \times (1 + \frac{k_{agl}^s}{100})$$

$$M_{agl}^t = \begin{cases} m_{agl}^t \geq 0 : M_{agl}^t = m_{agl}^t \times (1 + \frac{k_{agl}^m}{100}) \\ m_{agl}^t < 0 : M_{agl}^t = m_{agl}^t \times (1 - \frac{k_{agl}^m}{100}) \end{cases}$$

Where: f_{al}^{t} is the deterministic fertility rate for women of age *a* in location (TLA) *l* between time *t* and time *t*+1 estimated and extrapolated from data;

k_{al}^{f} is the fertility parameter that is drawn from a probability distribution;

s_{agl}^{t} is the deterministic survivorship rate for people of age *a* and gender *g* in location (TLA) *l* between time *t* and time *t*+1 estimated and extrapolated from data;

k_{agl}^{s} is the mortality parameter that is drawn from a probability distribution;

m_{agl}^{t} is the net migration rate of people of age *a* and gender *g* to/from location *l* between time *t* and time *t*+1 estimated and extrapolated from data;

k_{agl}^{m} is the migration parameter that is drawn from a probability distribution; and subscript *l* referring to the selected TLA in the Waikato region.

In the set of stochastic projections discussed in Section 6, the parameters are further simplified by assuming that $k_{al}^{f} = k^{f}$ for all *a* and *l*; $k_{agl}^{s} = k^{s}$ for all *a*, *g* and *l*; and $k_{agl}^{m} = k^{m}$ for all *a*, *g* and *l*. k^{f}, k^{s} and k^{m} are drawn from independently distributed normal distributions.

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